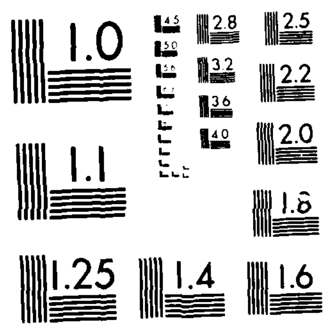
AD-A164 041 NUMERICAL SOLUTIONS OF SELECTED PROBLEMS WHICH ARISE IN THE THEORY OF PAR..(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI.. J A NOBLEY 1/2

UNCLASSIFIED DEC 85 AFIT/GAE/MA/85D-1 F/G 20/7 NL



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

NUMERICAL SOLUTIONS OF SELECTED PROBLEMS WHICH ARISE IN THE THEORY OF PARTICLE BEAMS

THESIS

James A. Mobley
Captain, USAF

AFIT/GAE/MA/85D-1

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

AFIT/GAE/MA/85D-1

7

NUMERICAL SOLUTIONS OF SELECTED
PROBLEMS WHICH ARISE IN THE
THEORY OF PARTICLE BEAMS

THESIS

James A. Mobley
Captain, USAF

AFIT/GAE/MA/85D-1

AFIT/GAE/MA/85D-1

NUMERICAL SOLUTIONS OF SELECTED PROBLEMS WHICH ARISE IN THE THEORY OF PARTICLE BEAMS

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Masters of Science in Aerospace Engineering

James A. Mobley, B.A.E.
Captain, UASF

December 1985

Approved for public release; distribution unlimited

Preface

The objectives of this study was to verify the closed form solution developed in reference [1] and to investigate a finite difference solution to a higher order dynamics model.

In completing this work I have had an enormous amount of help from my class mates who are to numerable to mention here. However, I would like to specifically thank Captain Netzer for his steady hand and his expert advise which was sincerely appreciated. Also I extend my deepest thanks and appreciation to Dr. Dennis Quinn, my thesis advisor, who kept me on the straight and narrow road to completion. Finally I must express my gratitude to my family who have been so terribly neglected through out these last eighteen months.

Table of Contents

List of Figures

List of Tables

Table | Page

List of Tables

LIST OF SYMBOLS

$N(r,t)$ - - - - - Particle density

$V_r(r,t)$ - - - - - Radial velocity

$V_\theta(r,t)$ - - - - - Angular velocity

$V_z(r,t)$ - - - - - Axial velocity

$E_r(r,t)$ - - - - - Radial electric field

$\delta N(r,t)$ - - - - - Perturbed number density

$\delta V_r(r,t)$ - - - - - Perturbed radial velocity

$\delta V_\theta(r,t)$ - - - - - Perturbed angular velocity

$\delta V_z(r,t)$ - - - - - Perturbed axial velocity

$\delta E_r(r,t)$ - - - - - Perturbed radial electric field

$\delta E_r^e(r,t)$ - - - - - Perturbed external radial electric field

$\delta E_\theta^e(r,t)$ - - - - - Perturbed external angular electric field

$\delta E_z^e(r,t)$ - - - - - Perturbed external axial electric field

$\delta B_r^e(r,t)$ - - - - - Perturbed external radial magnetic field

$\delta B_\theta^e(r,t)$ - - - - - Perturbed external angular magnetic field

$\delta B_z^e(r,t)$ - - - - - Perturbed external axial magnetic field

I - - - - - Current

R_b - - - - - Radius of beam

M_o - - - - - Rest mass of particle

q - - - - - Charge

$\underline{X}(*)$ - - - - - Perturbed values

$S(*)$ - - - - - State transition matrix

$\underline{g}(*)$ - - - - - External input

<u>List of Symbols</u> (continued)

W^o	-----	Initial state of W(t)
N^o	-----	Initial state of N(r,t)
r	-----	Radial variable
t	-----	Time (as a variable)
W	-----	Angular frequency
W_p	-----	Plasma frequency
W_c	-----	Cyclotron frequency
τ	-----	Constant defined in chapter 2 (See equation 2.7a)
C	-----	Velocity of light
π	-----	Pi
Ω	-----	Frequency (See equation 2.8)
K.E.	-----	Kinetic energy
R.M.E.	-----	Rest mass energy
B_o	-----	External axial magnetic field
U_o	-----	Permittivity of free space, $4\pi \times 10^{-7}$
E_o	-----	Permittivity of free space, $10^{-9}/36\pi$
a_{23}	-----	Constant defined in chapter 2 (See equation 2.7k)
a_{24}	-----	Constant defined in chapter 2 (See equation 2.7e)
a_{25}	-----	Constant defined in chapter 2 (See equation 2.7f)
a_{52}	-----	Constant defined in chapter 2 (See equation 2.7g)
$V_z{}^o$	-----	Equilibrium axial velocity

List of Symbols (continued)

D_r ----- $\frac{\partial}{\partial r}$ (f(r))

$\tilde{D}_r$ ----- (1/r) $\frac{\partial}{\partial r}$ (r f(r))

Abstract

A previous study at the Air Force Institute of Technology has led to the development of a closed form solution to the dynamic behavior of a charged particle beam. This solution was found for a simple, but nontrivial, single degree of freedom model (the "electrostatic approximation model"). This study investigates the characteristics of this closed form solution using three different types of inputs. The closed form solution is then verified by using finite differences to generate compatible results. Also investigated are the characteristics of the dynamic model prior to the implementation of the electrostatic approximation.

CHAPTER 1

1-1 Introduction.

In Reference [1] a system of partial differential equations is presented as a description of the dynamic behavior of a particle beam. This system is then cast as an abstract Cauchy problem. This results in a set of equations in the following form:

$$(d/dt)\,\underline{X} = A\underline{X} + \underline{g}(t) \qquad (1.1)$$

Where A is a non-constant 9x9 matrix and $\underline{g}(t)$ is the input vector.

In [1], this problem is analyzed using semigroup theory, from which a closed form solution is derived for a single degree of freedom model (the "Electrostatic Approximation Model").

In this thesis, numerical solutions are obtained using this closed form solution. Also an alternative solution of the Electrostatic Approximation Model is derived as is an approximate solution of the full system. See Figure 1 for a block diagram representation of the overall flow of this thesis. Section 1-2 describe in more detail the contents of the following chapters.

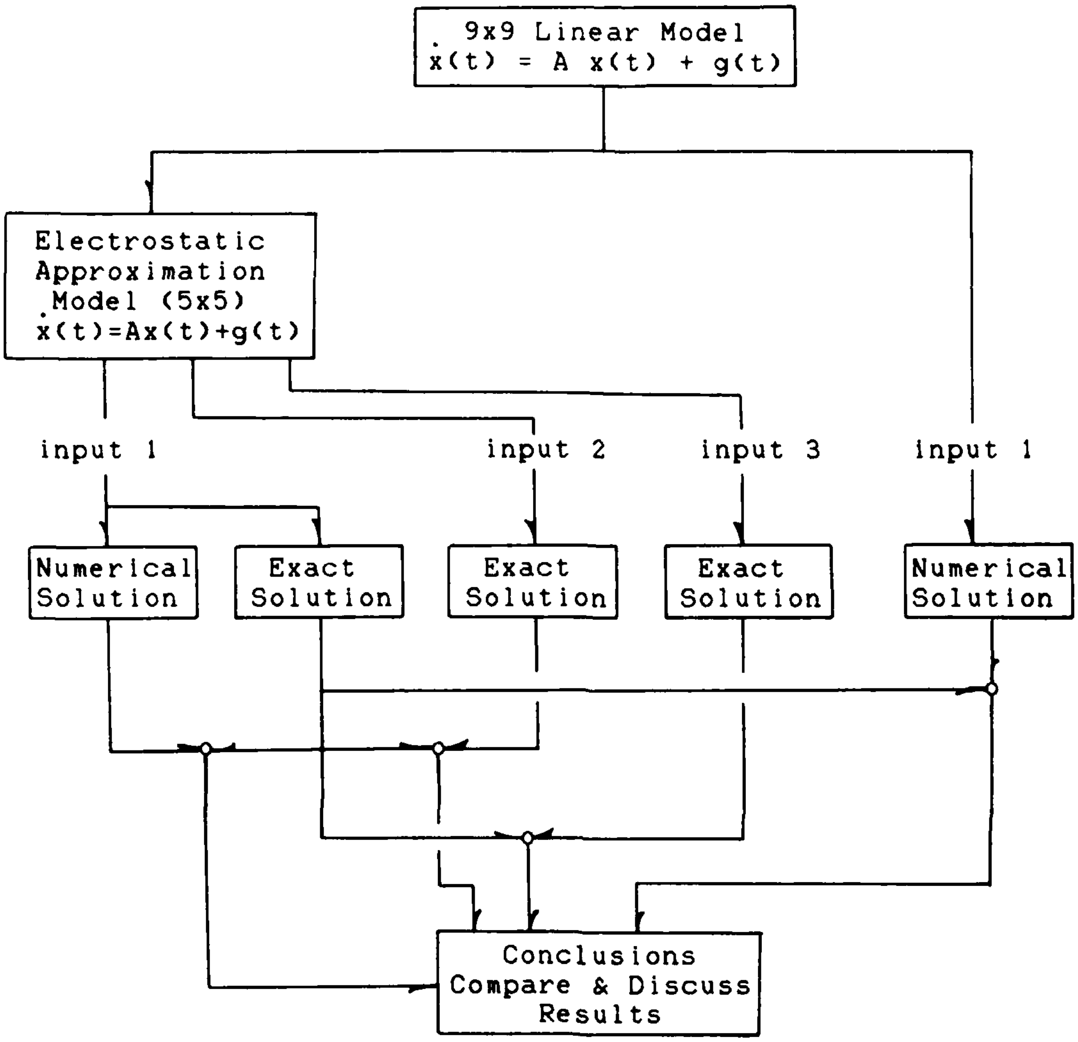


Figure 1. Investigation Flow Chart

1-2 Chapter Descriptions

In Chapter 2 of this thesis the closed form solution discussed in Section 1-1 is investigated using three distinct types of inputs. The first input is a step function which is implemented at t = 0.0. The second input is a function which exponentially approaches its maximum value. The third input rapidly reaches its maximum value, is constant for an interval of time and then rapidly returns to equilibrium.

In Chapter 3, the system of equations discussed in Section 1-1, Equation (1.1), are solved numerically after the system is reduced to fifth order by means of the Electrostatic Approximation Model. By approximating the partial derivitives in r with finite difference quotients and then solving the resulting differential equations, the solutions are determined. Since four of the five equations in this system do not involve derivatives with respect to r, they are solved exactly. The input used in this chapter is the step input described earlier.

The analysis presented in Chapter 4 is closely related to the Chapter 3 material with the Electrostatic Approximation Model not being used. Therefore, the system shown in Section 1-1, Equation (1.1) is of ninth-order. Also, this system of differential equations are even more complex since five of the nine equations involve finite

difference approximations. The same input to the system used in Chapter 3 is also used for Chapter 4, however the solution technique is considerably different due to the complexity of the problem. This ninth order system of differential equations is solved by an IMSL routine called DGEAR using the Cyber Computer System.

Chapter 5 of this thesis contains conclusions and recommendations along with all comparisons of data and discussion of results.

CHAPTER 2

2-1 Introduction.

The purpose of this section is to use the closed form solution to the reduced order, linear, single degree of freedom model found in Reference [1: page IV-21] for a given input and calculate the perturbed state of the particle beam in question. The results from this section are used later for comparison with results from other procedures.

The particle beam used throughout this effort is assumed to be at equilibrium for $t < 0$. In order to meet this assumption, an external magnetic field (B_o) with only the Z component non zero is applied for all time (t). The calculations required to determine B_o are found in Section 2-3. The beam parameters are as follows:

Current (I) = 1 amperes (amps)

Radius (Rb) = 10 millimeters (mm)

Kinetic Energy (KE) = 10 million electron volts (mev)

Mass (M_o) = $1.6726825 \times 10^{-27}$ kilograms (kg)

Charge (q) = 1.60219×10^{-19} coulomb

2-2 Setting Up The Problem.

In Reference [1: page I-1] a set of partial differential equations that describe the dynamic behavior of a particle beam are cast into the form of an abstract Cauchy problem. The ninth-order linear system of equations is of the form

$$(d/dt) \underline{X}(t) = A \underline{X}(t) + g(t) \qquad (2.1a)$$

where $\underline{X}(t)$ is the perturded values from equilibrium and g(t) and A are defined as follows:

$$g(t) = a_{25} \begin{bmatrix} 0 \\ \delta E_r^e(r,t) - W\, r\, \delta B_z^e(r,t) - V_z^o\, \delta B_\theta^e(r,t) \\ \delta E_\theta^e(r,t) + V_z^o\, \delta B_r^e(r,t) \\ \delta E_z^e(r,t) + W\, r\, \delta B_r^e(r,t) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (2.1b)$$

$$A = \begin{bmatrix} 0 & -N^o \tilde{D}_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{23} & a_{24}\, r & a_{25} & 0 & 0 & -a_{25} V_z^o & -a_{25} W r \\ 0 & -a_{23} & 0 & 0 & 0 & a_{25} & 0 & 0 & 0 \\ 0 & -a_{24}\, r & 0 & 0 & 0 & 0 & a_{25} & 0 & 0 \\ 0 & a_{52} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{61} & 0 & a_{52} & 0 & 0 & 0 & 0 & 0 & -C^2 D_r \\ a_{71} & 0 & 0 & a_{52} & 0 & 0 & 0 & C^2 \tilde{D}_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & D_r & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\tilde{D}_r & 0 & 0 & 0 \end{bmatrix} \tag{2.1c}$$

where

$$a_{61} = -\left(a_{52}\, W\, r \right) / N^o$$

$$a_{71} = \left(a_{52}\, V_z^o \right) / N^o$$

and a_{23}, a_{24}, a_{25}, a_{52}, $\tilde{D}_r$, D_r, are referenced in the symbols page and defined in Equations (2.7a) to (2.7k) Next, this system of equations can be reduced by the implementation of the Electrostatic Approximation Model described in Chapter 1 and fully developed in Reference [1: page III-39]. This approximation reduces A and g(t) of Equation (2.1) to the following much more manageable system.

$$A = \begin{bmatrix} 0 & -N^o \tilde{D}_r & 0 & 0 & 0 \\ 0 & 0 & a_{23} & a_{24}\, r & a_{25} \\ 0 & -a_{23} & 0 & 0 & 0 \\ 0 & -a_{24}\, r & 0 & 0 & 0 \\ 0 & a_{52} & 0 & 0 & 0 \end{bmatrix} \qquad (2.2a)$$

$$g(t) = a_{25} \begin{bmatrix} 0 \\ \delta E_r^e(r,t) - W\, r\, \delta B_z^e(r,t) - V_z^o\, \delta B_\theta^e(r,t) \\ \delta E_\theta^e(r,t) + V_z^o\, \delta B_r^e(r,t) \\ \delta E_z^e(r,t) + W\, r\, \delta B_r^e(r,t) \\ 0 \end{bmatrix} \qquad (2.2b)$$

The closed form solution found in Reference [1: page

IV-21] to this fifth order system of equations and mentioned in the previous section is as follows.

$$\underline{X}(t) = S(t)\underline{X}^{o} + \int_{o}^{t} S(t-s)g(s)ds \qquad (0 < t < T) \qquad (2.3)$$

where

$$\underline{X}(t) = \begin{bmatrix} \delta N(r,t) \\ \delta V_r(r,t) \\ \delta V_\theta(r,t) \\ \delta V_z(r,t) \\ \delta E_r(r,t) \end{bmatrix}$$

g(s) is as defined in Equation (2.2) where t is replaced by s, and, from Reference [1: page IV-22], the state transition matrix S(t) is the following 5x5 system

$$
S(t) = \begin{bmatrix}
1 & -N^{o}\,\tilde{D}_r \dfrac{\sin[\Omega t](.)}{\Omega} & N^{o}\,\tilde{D}_r \dfrac{(\cos\,[\Omega t]-1)}{\Omega^2} a_{23}\,(.) & N^{o}\tilde{D}_r \dfrac{(\cos[\Omega t]-1)}{\Omega^2} a_{24}\, r\,(.) & N^{o}\tilde{D}_r \dfrac{(\cos[\Omega t]-1)}{\Omega^2} a_{25}\,(.) \\
0 & \cos\,[\Omega t] & \dfrac{a_{23}\sin\,[\Omega t]}{\Omega} & \dfrac{a_{24}\, r \sin[\Omega t]}{\Omega} & \dfrac{a_{25}\sin[\Omega t]}{\Omega} \\
0 & \dfrac{-a_{23}\sin\,[\Omega t]}{\Omega} & 1+\left[\dfrac{a_{23}}{\Omega}\right]^2 (\cos[\Omega t]-1) & \dfrac{a_{23}\, a_{24}\, r}{\Omega^2}(\cos[\Omega t]-1) & \dfrac{a_{23}\, a_{25}}{\Omega^2}(\cos[\Omega t]-1) \\
0 & \dfrac{-a_{24}\, r \sin\,[\Omega t]}{\Omega} & \dfrac{a_{24}\, r\, a_{23}}{\Omega^2}(\cos[\Omega t]-1) & 1+\left[\dfrac{a_{24}\, r}{\Omega}\right]^2(\cos[\Omega t]-1) & \dfrac{a_{24}\, a_{25}\, r}{\Omega}(\cos[\Omega t]-1) \\
0 & \dfrac{a_{52}\sin\,[\Omega t]}{\Omega} & \dfrac{-a_{52}\, a_{23}}{\Omega^2}(\cos[\Omega t]-1) & -\dfrac{a_{24}\, a_{52}\, r}{\Omega^2}(\cos[\Omega t]-1) & 1-\dfrac{a_{25}\, a_{52}}{\Omega^2}(\cos[\Omega t]-1)
\end{bmatrix} \quad (2.4)
$$

$\underline{X}^{o}$ is the initial state of $\underline{X}(t)$ and hence is the difference between the initial state of the beam and the equilibrium. Since, in the problem stated above, these quantities are equal,

$$\underline{X}^{o} = \underline{0} \tag{2.5}$$

Equation (2.3) then becomes

$$\underline{X}(t) = \int_{0}^{t} S(t-s)g(s)ds \quad \text{for } (0 < t < T) \tag{2.6}$$

It should be noted that the external magnetic field required to achieve equilibrium is not included in the input term g(s). This is because the formulation of the problem leads to the solution of Equation (2-3) in which perturbed quantities $\underline{X}(t)$ depend only on perturbed inputs δE_r^{e} through δB_r^{e} in g(s) while equilibrium quantities appear in S(t).

2-3 Calculation of the Entries of S(t-s) and g(s).

Now, to calculate the entries of S(t-s) and g(s) of Equation (2.6), τ, V_z^o, N_o, W_p, a_{24}, a_{25}, a_{52} B_o, W_c, W, a_{23}, and Ω must first be determined. These quantities are defined in the list of symbols pages viii to x and are now numerically calculated.

2.3.1 Calculations of τ, V_z^o, N^o, W_p

From Reference [2: page 2], if

$$\tau = 1 + \frac{K.E.}{R.M.E.} \tag{2.7a}$$

$$= 1.01065$$

then

$$V_z^o = (1 - 1/\tau^2)^{1/2} C \tag{2.7b}$$

$$= 4.34198 \times 10^7 \text{ m/sec}$$

For the remainder of this thesis the beam is assumed to be non-relativistic and τ is assumed to be equal to one.

From Reference [2: page 1] assuming uniform current density

$$N^o = I/(\pi R_b^2 q V_z^o) \qquad (2.7c)$$

$$= 4.57559 \times 10^{14} \ 1/m^3$$

From Reference [2: page 3]

$$W_p^2 = \frac{q^2}{M_o} (U_o C^2 N^o) \qquad (2.7d)$$

so

$$W_p = 2.81613 \times 10^7 \text{ rad/sec}$$

2-3.2 Calculations of a_{24}, a_{25}, a_{52}, B_o

From Reference [1: page III-37]

$$a_{24} = \frac{-Wp^2 V_z^o}{2 C^2} \qquad (2.7e)$$

$$= -1.91571 \times 10^5 \ (m\text{-sec})^{-1} ,$$

$$a_{25} = \frac{q}{M_o} \qquad (2.7f)$$

$$= 9.57856 \times 10^7 \text{ coulomb/kg} ,$$

and

$$a_{52} = -U_o\ q\ C^2\ N^o \qquad (2.7g)$$

$$= -8.27953 \times 10^6 \ \text{kg/sec}^2\text{-coulomb}$$

From Reference [2: page 3]

$$B_o{}^2 = \frac{2\ M_o\ I}{q\ \tau\ \pi\ \mathcal{E}_o\ V_z{}^o\ Rb^2} \qquad (2.7h)$$

so

$$B_o = .41578 \ \text{tesla}$$

2-3.3 <u>Calculations of Wc, W, a_{23}, Ω</u>

From Reference [2: page 3], Wc is calculated by

$$Wc = \frac{q\ B_o}{M_o} \qquad (2.7i)$$

$$= 3.98261 \times 10^7 \ \text{rad/sec} \ ,$$

From Reference [2: page 3]

$$W_\theta = -W_c\ (1 \pm (1 - 2\ W_p{}^2\ /\ W_c{}^2)^{1/2})\ \ /2 \qquad (2.7j)$$

It is important to note here that W_θ found above is

related to W of Reference [1: page III-27] by the following relationship.

$$W = -W_e$$

The value for W is used from this point on. This is done to comply with the nomenclature found in Reference [1: page III-37].

In Reference [1: page III-37], a_{23} is defined to be

$$a_{23} = W_c - 2\,W \qquad (2.7k)$$

One type of flow which is often considered is Brillouin flow (*see Reference [2: page 4: case 2]*). For Brillouin flow and using the definitions found above, W is calculated to be

$$W = W_c \,/\, 2 \qquad (2.7l)$$

$$\approx 19913067.12803 \text{ rad/sec}$$

Thus, for Brillouin flow

$$a_{23} \approx 0.0 \text{ rad/sec}$$

A key frequency which is used later is denoted by the symbol

Ω and is defined by

$$\Omega = [\, Wc^2 - Wp^2(\, 1 - A\,)\,]^{1/2} \qquad (2.8)$$

where : $A = Wp^2 \left[\dfrac{V_z^o\ r}{2\ C^2} \right]^2$

For the values of r, V_z^o, and C,

A is sufficiently close to zero to be considered negligible.

Moreover, $W_c^2 = 2\ W_p^2$.

Therefore Equation (2.8) becomes

$$\Omega \approx Wp$$

$$= 2.81613 \times 10^7 \text{ rad/sec} \quad 2\text{-}4$$

2-4 Evaluating g(s),S(t-s)g(s), and X(t).

At this point all the terms necessary to completely determine a solution to Equation (2.1) starting with A and g(s) defined in Equation (2.2) are available.

To maintain the rigid rotor equilibrium, the only external field which is non-zero is the Z component of the

external magnetic field (B_o). For this thesis, the perturbed input is a small perturbation of this external field and consequently is taken to be one percent of B_o.

Therefore the input g(s) reduces to :

$$g(s) = a_{25} \begin{bmatrix} 0 \\ -W\, r\, \delta B_z^{e}(r,s) \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.9}$$

$$= a_{25} \begin{bmatrix} 0 \\ (\ -82795.32170\ \mathrm{kg/coulomb - sec^2}\ )\ r \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The next step is to find the product of S(t-s)g(s) and perform the required integration.

Using the definition of S(t-s) and g(s):

$$S(t-s)g(s) = a_{25} \begin{bmatrix} -N^{o}\ \tilde{D}r\ \dfrac{\sin[\ \Omega(t-s)\ ]}{\Omega}\ r\ (\ D\ ) \\ \cos[\ \Omega(t-s)\ ]\ r\ (\ D\ ) \\ \dfrac{-a_{23}\ r\ \sin[\ \Omega(t-s)\ ]\ (\ D\ )}{\Omega} \\ \dfrac{-a_{24}\ r^{2}\ \sin[\ \Omega(t-s)\ ]\ (\ D\ )}{\Omega} \\ \dfrac{a_{52}\ r\ \sin[\ \Omega(t-s)\ ]\ (\ D\ )}{\Omega} \end{bmatrix} \qquad (2.10)$$

where D = -82795.32170 kg/coulomb-sec^2

Now, to perform the required integration necessary to determine $\underline{X}(t)$ in Equation (2.7), the trigonometric functions of Equation (2.10) can be expanded by angle sum and angle difference relations. After this expansion is completed and constants are factored out of the integral the resulting values can be determined. They are as follows:

$$
\underline{X}(t) = a_{25} \begin{bmatrix} \dfrac{-2 \quad D\ N^{o}}{\Omega^{2}} \left(1 - \cos[\ \Omega t\ ] \right) \\ \dfrac{D\ r}{\Omega} \ \sin[\ \Omega t\ ] \\ \dfrac{-\ D\ a_{23}\ r}{\Omega^{2}} \left(1 - \cos[\ \Omega t\ ] \right) \\ \dfrac{-\ D\ a_{24}\ r^{2}}{\Omega^{2}} \left(1 - \cos[\ \Omega t\ ] \right) \\ \dfrac{-\ D\ a_{52}\ r}{\Omega^{2}} \left(1 - \cos[\ \Omega t\ ] \right) \end{bmatrix} \qquad (2.11)
$$

and, substituting the appropriate values for the constants gives:

$$
\underline{X}(t) = \begin{bmatrix} (\ 9151.20373\ 1/\mathrm{mm}^{3}\ ) * (\ 1 - \cos[\ \Omega t\ ]\ ) \\ (\ -281613.48000\ \ 1/\mathrm{sec}\ ) * \ r * \sin[\ \Omega t\ ] \\ (\ 0.00\ ) \\ (-1915.71291\ 1/\mathrm{m\text{-}sec}\ ) * (\ r^{2})) * (\ 1 - \cos[\ \Omega t\ ]) \\ (\ 82795.44549\ \mathrm{kg/sec^{2}\text{-}coulomb}\ ) * (\ r\ ) * (\ 1 - \cos[\ \Omega t\ ]) \end{bmatrix}
$$

The above results are tabulated in Tables 1 thru 4 and are shown graphically in Figures 2 thru 5.

2-5 Inputs Other than Constant

In this section the work presented in Section 2-4 is repeated, only this time using different external inputs $\delta B_z^e(r,t)$. The first of these inputs is designed to more realistically model how an actual input would be applied to the system. In other words, the input reaches its maximum value rapidly instead of instantaneously as assumed in Section 2-4, and as indicated in Figure 18. This work is presented in Section 2-5.1. The second input explored in this section is a fluctuation from the equilibrium value of $\delta B_z^e(r,t)$ which rapidly reaches its maximum. Then, after a specified amount of time (t_1) it decreases back to its equilibrium value (see Figure 19). The input for this third case is the same as that in Figure 18 for $0 \leq t \leq 6 \times 10^{-09}$ seconds. It is then constant for $6 \times 10^{-09} \leq t \leq 444 \times 10^{-09}$ seconds. The input then decreases as indicated in Figure 19 for $444 \times 10^{-09} < t < 450 \times 10^{-09}$ seconds. For $t > 450 \times 10^{-09}$ seconds the input is zero.

2-5.1 Rapid but Continuous Increase to Constant

Using the new input described in Section 2-5 the g(s) matrix becomes:

$$g(s) = a_{25} \begin{bmatrix} 0.0 \\ -W\ r\ \delta B_z^{e}(r,s) \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

where

$$\delta B_z^{e}(r,s) = .01\ B^{o}\ (\ 1 - \exp(\ -H\ s\ )\ )$$

and

$$H = 1 \times 10^{09}$$

The dynamic behavior of this new input is shown in Figure 18.

The last step required before integration is to find the product of the S(t-s) and g(s) matrices, where the S(t-s) matrix is exactly the same as used in Section 2-4 and the g(s) matrix is as defined above. This resulting product is

$$
S(t-s)g(s) = a_{25} \begin{bmatrix} N^o \tilde{D}_r \dfrac{\sin[\ \Omega(t-s)\ ]}{\Omega} (\ r\ W\ \delta B_z^e(r,s)\ ) \\ -\cos[\ \Omega(t-s)\ ]\ \ W\ r\ \delta B_z^e(r,s) \\ \dfrac{a_{23} \sin[\ \Omega(t-s)\ ]\ \ W\ r\ \delta B_z^e(r,s)}{\Omega} \\ \dfrac{a_{24}\ r\ \sin[\ \Omega(t-s)\ ]\ \ W\ r\ \delta B_z^e(r,s)}{\Omega} \\ \dfrac{-a_{52} \sin[\ \Omega(t-s)\ ]\ \ W\ r\ \delta B_z^e(r,s)}{\Omega} \end{bmatrix}
$$

Now, rearranging the terms of the S(t-s) g(s) matrix yields

$$
S(t-s)\ g(s) = a_{25} \begin{bmatrix} 2\ W\ N^o \dfrac{\sin[\ \Omega(t-s)\ ]}{\Omega} \delta B_z^e(r,s) \\ -\ W\ r\ \cos[\ \Omega(t-s)\ ]\ \delta B_z^e(r,s) \\ a_{23}\ \ W\ r \dfrac{\sin[\ \Omega(t-s)\ ]}{\Omega} \delta B_z^e(r,s) \\ a_{24}\ r^2\ W \dfrac{\sin[\ \Omega(t-s)\ ]}{\Omega} \delta B_z^e(r,s) \\ -a_{52}\ W\ r \dfrac{\sin[\ \Omega(t-s)\ ]}{\Omega} \delta B_z^e(r,s) \end{bmatrix}
$$

Before performing the required integration on S(t-s)g(s) it is important to notice that this requires the integration of a product of the form:

$$
\sin[\ \Omega(t-s)\ ]\ \exp(\ -a(s)\ )
$$

This type of integration can be performed by first

expanding the trigometric functions using angle sum and angle difference relations and then integrating the individual terms using standard integration formulae. Therefore, the results for Equation (2.6) are as follows:

$$\delta N(r,t) = \frac{(.02)\ B^{o}\ a_{25}\ W\ N^{o}\ (\ X + Y - Z\ )}{\Omega} \qquad (2.12)$$

where $X = \dfrac{1 - \cos \Omega t}{\Omega}$

$$Y = \exp(\ -H\ t\ )\ \left[\frac{H \sin[\ 2\Omega t\ ] + \Omega \cos[\ 2\Omega t\ ]}{(\ -H\ )^2 + \Omega^2}\right]$$

$$Z = \frac{H \sin[\ \Omega t\ ] + \Omega \cos[\ \Omega t\ ]}{(\ -H\ )^2 + \Omega^2}$$

$$\delta V_r(r,t) = -a_{25}\ W\ r\ (.01)\ B^{o}\ (\ X2 + Y2 + Z2\ ) \qquad (2.13)$$

where $X2 = \dfrac{\sin[\ \Omega t\ ]}{\Omega}$

$$Y2 = \frac{H \exp(\ -H\ t\ )}{(\ -H\ )^2 + \Omega^2}$$

$$Z2 = \frac{-H\ \cos[\ \Omega t\ ] - \Omega \sin[\ \Omega t\ ]}{(\ -H\ )^2 + \Omega^2}$$

$$\delta V_{\phi}(r,t) = (.01)\ B^{o}\ a_{23}\ a_{25}\ W\ r\ (\ X3 + Y3 - Z3\ ) \qquad (2.14)$$

where $X3 = \dfrac{1 - \cos[\ \Omega t\ ]}{\Omega}$

$$Y3 = \exp(\ -H\ t\ )\ \left[\frac{H \sin[\ 2\Omega t\ ] + \Omega \cos[\ 2\Omega t\ ]}{(\ -H\ )^2 + \Omega^2}\right]$$

$$Z3 = \frac{H \sin[\ \Omega t\ ] + \Omega \cos[\ \Omega t\ ]}{(\ -H\ )^2 + \Omega^2}$$

$$\delta V_z(r,t) = \frac{a_{24}\ a_{25}\ r^2\ W\ (.01)\ B^o}{\Omega}\ (\ X4 + Y4 - Z4\ ) \qquad (2.15)$$

where
$$X4 = \frac{1-\cos[\ \Omega t\ ]}{\Omega}$$

$$Y4 = \exp(\ -H\ t\ ) \left[\frac{H \sin[\ 2\Omega t\ ] + \Omega \cos[\ 2\Omega t\ ]}{(\ -H\ )^2 + \Omega^2} \right]$$

$$Z4 = \frac{H \sin[\ \Omega t\ ] + \Omega \cos[\ \Omega t\ ]}{(\ -H\ )^2 + \Omega^2}$$

$$\delta E_r(r,t) = \frac{-a_{52}\ a_{25}\ W\ r\ (.01)\ B^o}{\Omega}\ (\ X5 + Y5 - Z5\ ) \qquad (2.16)$$

where
$$X5 = \frac{1-\cos[\ \Omega t\ ]}{\Omega}$$

$$Y5 = \exp(\ -H\ t\ ) \left[\frac{H \sin[\ 2\Omega t\ ] + \Omega \cos[\ 2\Omega t\ ]}{(\ -H\ )^2 + \Omega^2} \right]$$

$$Z5 = \frac{H \sin[\ \Omega t\ ] + \Omega \cos[\ \Omega t\ ]}{(\ -H\ )^2 + \Omega^2}$$

The results from this section are tabulated in Tables 5 thru 8 and are shown graphically in Figures 6 thru 9.

2-5.2 Rapid Increase to Constant Followed by a Rapid Decrease to Zero

The third and final type of external input $\delta B_z^e(r,t)$ explored in this chapter is a fluctuation away from zero which returns to zero. The procedure presented in Section 2-5.1 is repeated using this new input. Since the procedures are nearly identical, the work presented in this section is without explanation. If a question arises from the analysis, refer to Section 2-5.1 for an explanation.

The input is defined by

$$\delta B_z^e(r,t)= \begin{cases} 0.01\ B^o\ (1-e^{-H\ t}) & 0<t<t_1 \quad (2.17a) \\ 0.01B^o \left[1- \dfrac{3(t-t_1)^2}{(t_2-t_1)^2} + \dfrac{2(t-t_1)^3}{(t_2-t_1)^3}\right] & t_1<t<t_2 \quad (2.17b) \\ 0.0 & t>t_2 \end{cases}$$

where t_1 = 4.45 E - 7

t_2 = 4.50 E - 7

Using this definition for the external input requires that the integration of the S(t-s) g(s) matrix be performed over two intervals of time with each interval having a different integrand which involves the external input as shown in the above equation. The integration over the first

interval has already been done in Section 2-5.1. Therefore, all that remains is to perform the integration over the second time interval and add the solution found in Section 2-5.1. The integration of the S(t-s) g(s) matrix over the second time interval results in the following:

$$\delta N(r,t) = \frac{0.02\ B_o\ a_{25}\ W\ N^o}{\Omega} \left[\frac{1 - \cos[\Omega(t-t_1)]}{\Omega} - \frac{3}{(t_2-t_1)^2} \left[\frac{\Omega^2\ (t-t_1) - 2}{\Omega^3} + \frac{2 \cos[\Omega(t-t_1)]}{\Omega^3} \right] + \frac{2}{(t_2-t_1)^3} \left[\frac{\Omega^2\ (t-t_1)^3 - 6\ (t-t_1)}{\Omega^3} + \frac{6 \sin[\Omega(t-t_1)]}{\Omega^4} \right] \right] \tag{2.18}$$

$$\delta V_r(r,t) = -0.01\ B_o\ a_{25}\ W\ r \left[\frac{\sin[\ \Omega(t-t_1)\ ]}{\Omega} - \frac{3}{(t_2-t_1)^2} \left[\frac{2\ (t-t_1)}{\Omega^2} - \frac{2 \sin[\Omega(t-t_1)]}{\Omega^3} \right] + \frac{2}{(t_2-t_1)^3} \left[\frac{3\ \Omega^2\ (t-t_1)^2 - 6}{\Omega^4} + \frac{6 \cos[\Omega(t-t_1)]}{\Omega^4} \right] \right] \tag{2.19}$$

$$\delta V_\theta(r,t) = 0.01\ B_o\ a_{23}\ a_{25}\ W\ r \left[\frac{1 - \cos[\Omega(t-t_1)]}{\Omega} - \frac{3}{(t_2-t_1)^2} \left[\frac{\Omega^2\ (t-t_1) - 2}{\Omega^3} + \frac{2 \cos[\Omega(t-t_1)]}{\Omega^3} \right] + \frac{2}{(t_2-t_1)^3} \left[\frac{\Omega^2\ (t-t_1)^3 - 6\ (t-t_1)}{\Omega^3} + \frac{6 \sin[\Omega(t-t_1)]}{\Omega^4} \right] \right] \tag{2.20}$$

$$\delta V_z(r,t) = \frac{0.01\ B_o\ a_{25}\ a_{24}\ W\ r^2}{\Omega}\left[\frac{1 - \cos[\Omega(t-t_1)]}{\Omega}\right. \quad (2.21)$$

$$- \frac{3}{(t_2-t_1)^2}\left[\frac{\Omega^2\ (t-t_1) - 2}{\Omega^3} + \frac{2\cos[\Omega(t-t_1)]}{\Omega^3}\right]$$

$$\left. + \frac{2}{(t_2-t_1)^3}\left[\frac{\Omega^2\ (t-t_1)^3 - 6\ (t-t_1)}{\Omega^3} + \frac{6\sin[\Omega(t-t_1)]}{\Omega^4}\right]\right]$$

$$\delta E_r(r,t) = \frac{-0.01\ B_o\ a_{25}\ a_{52}\ W\ r}{\Omega}\left[\frac{1 - \cos[\Omega(t-t_1)]}{\Omega}\right. \quad (2.22)$$

$$- \frac{3}{(t_2-t_1)^2}\left[\frac{\Omega^2\ (t-t_1) - 2}{\Omega^3} + \frac{2\cos[\Omega(t-t_1)]}{\Omega^3}\right]$$

$$\left. + \frac{2}{(t_2-t_1)^3}\left[\frac{\Omega^2\ (t-t_1)^3 - 6\ (t-t_1)}{\Omega^3} + \frac{6\sin[\Omega(t-t_1)]}{\Omega^4}\right]\right]$$

The dynamic behavior of the perturbed quantities, $\underline{X}(t)$, are much more complicated to determine in this section due to the form of the external axial magnetic input. As shown by Equation (2.17), the solution is found over three distinct time intervals. Over the first interval, from $t = 0.0$ to $t = t_1$, the input and solution are the same as that determined in Section 2-5.1. For $t_1 < s < t_2$, the second interval, the input is defined by Equation (2.17). The solution is found by adding the results presented by Equations (2.18) thru (2.22) with the results from Section 2-5.1 with the integration preformed from $t=0.0$ to $t = t_1$.

Over the third interval, $t_2 < s < t$, the input is defined by Equation (2.17) as zero and the solution is the summation of the Section 2-5.1 material integrated from $t = 0.0$ to $t = t_1$ with Equations (2.18) thru (2.22) integrated from $t = t_1$ to $t = t_2$. The results are presented in Tables 9 thru 16 and are shown graphically in Figures 10 thru 13. They represent the results from this section after each particular type of input was applied to the system for its associated amount of time.

CHAPTER 3

3-1 Introduction.

This chapter presents an analysis of the linear, time invariant, partial differential equations that govern the dynamic behavior of the particle beam discussed in Chapter 2. However, in this chapter the solution to the governing equations is found numerically. This is done by approximating the partial derivitives in r with finite difference quotients and then evaluating the equations along the radius of the beam at selected points. These results are tabulated and compared to the results found in Chapter 2 from the closed form solution technique derived in Reference [1: page IV-21].

3-2 Setting Up the Problem.

From Reference [1: page III-40] the governing partial differential equations discussed above are as follows :

$$(d/dt) \; \underline{X}(t) =$$

$$\begin{bmatrix} 0 & -N^o \tilde{D}_r & 0 & 0 & 0 \\ 0 & 0 & a_{23} & a_{24}\, r & a_{25} \\ 0 & -a_{23} & 0 & 0 & 0 \\ 0 & -a_{24}\, r & 0 & 0 & 0 \\ 0 & a_{52} & 0 & 0 & 0 \end{bmatrix} \underline{X}(t) \tag{3.1}$$

$$+ \; a_{25} \begin{bmatrix} 0 \\ \delta E_r^{\,e}(r,t) - W\, r\, \delta B_z^{\,e}(r,t) - V_z^{\,o}\, \delta B_\theta^{\,e}(r,t) \\ \delta E_\theta^{\,e}(r,t) + V_z^{\,o}\, \delta B_r^{\,e}(r,t) \\ \delta E_z^{\,e}(r,t) + W\, r\, \delta B_r^{\,e}(r,t) \\ 0 \end{bmatrix}$$

where $\tilde{D}_r f = (1/r) \; D_r(r\, f(r))$ (3.2)

and where the values for a_{23}, a_{24}, a_{25}, and a_{52} are the same as defined in Chapter 2. Also, as in Chapter 2, the only external field applied is $\delta B_z^{\,e}(r,t)$ and its maximum value is assumed to be one percent of the equilibrium value B_o. The rest of the external fields are assumed equal to zero. Therefore, Equation (3.1) becomes :

$$(d/dt) \; \underline{X}(t) =$$

$$\begin{bmatrix} 0 & -N^o \tilde{D}_r & 0 & 0 & 0 \\ 0 & 0 & a_{23} & a_{24}\, r & a_{25} \\ 0 & -a_{23} & 0 & 0 & 0 \\ 0 & -a_{24}\, r & 0 & 0 & 0 \\ 0 & a_{52} & 0 & 0 & 0 \end{bmatrix} \underline{X}(t) \tag{3.3}$$

$$+ \; a_{25} \begin{bmatrix} 0 \\ -W\, r\, \delta B_z^{\,e}(r,t) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Now, by using Equation (3.2) in Equation (3.3), the following five differential equations result.

$$(d/dt)\ \delta N(r,t) = \frac{-N^{o}}{r} \left[\delta V_r(r,t) + r\ D_r\ (\delta V_r(r,t)) \right] \quad (3.4)$$

$$(d/dt)\ \delta V_r(r,t) = a_{23}\ \delta V_\theta(r,t) + a_{24}\ r\ \delta V_z(r,t) \quad (3.5)$$
$$+ a_{25}\ \delta E_r(r,t) - W\ r\ \delta B_z^{e}(r,t)$$

$$(d/dt)\ \delta V_\theta(r,t) = -a_{23}\ \delta V_r(r,t) \quad (3.6)$$

$$(d/dt)\ \delta V_z(r,t) = -a_{24}\ r\ \delta V_r(r,t) \quad (3.7)$$

$$(d/dt)\ \delta E_r(r,t) = a_{52}\ \delta V_r(r,t) \quad (3.8)$$

3-3 The Numerical Approximation.

Equations (3.4) through (3.8) are the linear, time invariant, differential equations that govern the dynamic behavior of the particle beam. However, only Equation (3.4) contains a partial derivitive with respect to r. This partial derivitive is replaced with its finite difference approximation (backward differences) prior to investigating the solutions to Equations (3.4) through (3.8). This finite difference approximation is as follows :

$$D_r(\delta V_r(r,t)) \approx (\delta V_r(r,t) - \delta V_r(r-h,t)) / h \quad (3.9)$$

Where h is the spacing between points. Therefore, Equation (3.4) becomes:

$$(d/dt)\ \delta N(r,t) = (-N^{o}/r)\left[\delta V_r(r,t) + (r/h)\left(\delta V_r(r,t) - \delta V_r(r-h,t)\right)\right] \quad (3.10)$$

3-4 <u>Solution Technique.</u>

Out of the five governing differential equations discussed earlier, only Equation (3.10) requires a finite difference approximation. Now, since none of the other four equations are coupled to Equation (3.10), their solutions can be found by applying standard differential equation techniques. The solutions to these four equations are as follows :

$$\delta V_r(r,t) = \frac{a_{25}\ a_{52}\ r\ \sin[\ \Omega t\ ]}{100\ \Omega} \quad (3.11)$$

$$\delta V_\theta(r,t) = \frac{a_{23}\ a_{25}\ a_{52}\ r}{100\ \Omega}\left(\cos[\ \Omega t\ ] - 1\right) \quad (3.12)$$

$$\delta V_z(r,t) = \frac{a_{24}\ a_{25}\ a_{52}\ r^2}{100\ \Omega}\left(\cos[\ \Omega t\ ] - 1\right) \quad (3.13)$$

$$\delta E_r(r,t) = \frac{-\ a_{52}^{2}\ a_{25}\ r}{100\ \Omega}\left(\cos[\ \Omega t\ ] - 1\right) \quad (3.14)$$

where

$$\Omega = \left(a_{23}{}^2 + a_{24}{}^2 (r^2) - a_{52}\, a_{25} \right)^{1/2}$$

Note that this yields the same values for Ω as does Equation (2.8).

It should be noted that in the solution of these four equations, the initial values are zero. This is consistent with the derivation of the perturbation equations and agrees with the closed form solution found in Chapter 2 for t=0.

The solution of Equation (3.10), however, is slightly more difficult. Since Equation (3.10) contains finite differences and is coupled to the other four equations, the equation must be analyzed at specific points along the radius of the beam.

Now, after substituting in Equation (3.11), Equation (3.10) becomes

$$(d/dt)\ \delta N(r,t) = \frac{-N_o}{r} \left[\frac{a_{25}\, a_{52}\, r}{100\, \Omega} \sin[\, \Omega t\,] + r \frac{a_{25}\, a_{52}\, r \sin[\, \Omega t\,]}{100\, h\ \ \Omega} - \frac{a_{25}\, a_{52}\, (r-h) \sin[\, \Omega 1\ t\,]}{100\, h\ \ \Omega 1} \right] \tag{3.15}$$

where

$$\Omega 1 = (a_{23}{}^2 + a_{24}{}^2 (r - h)^2 - a_{52}\, a_{25})^{1/2}$$

Now the solution of Equation (3.16) can be found through simple integration, where the resulting constant of integration is determined by using the fact that $\delta N(r,t)$ is zero at t=0. Therefore, the solution of Equation (3.15) is:

$$\delta N(r,t) = \frac{-N^o}{} \left[\frac{a_{25}\, a_{52} \quad (1 - \cos[\, \Omega t\,])}{100\, \Omega} \right. \qquad (3.16)$$

$$\left. \frac{r\, (1 - \cos[\, \Omega t\,])}{h\, \Omega 1} - \frac{(r-h)\, (1 - \cos[\, \Omega 1\, t\,])}{h\, \Omega 1} \right]$$

We now have the solutions to all five governing differential equations.

3-5 Data Presentation.

In order to compare the results from Chapter 3 with those found in Chapter 2, the radius of the partical beam is divided into twenty evenly spaced intervals. Since the beam radius was given as ten millimeters, the spacing (h) is one half of a millimeter. For each value of time chosen there will be nineteen results for the five equations derived in Section 3-4. Each of these results represents a

solution at a specific radial distance from the center of the beam. The same format will be used for the data presented from Chapter 2.

The results from this chapter are presented in Table 17 thru 20 and are shown graphically in Figures 14 thru 17.

CHAPTER 4

4-1 Introduction

The material presented in this chapter is an expansion of that presented in Chapter 3 for the system (Equation 2.1) which results when the Electrostatic Approximation Model is not used to reduce the complexity of the system. The resulting ninth-order system is solved and compared with the results of Chapter 3 in order to evaluate the effect of the Electrostatic Approximation.

4-2 Setting up the Problem

The ninth-order system of governing differential equations are

$$(d/dt)\ \underline{X}(t) = A\ \underline{X}(t) + g(t) \qquad (4.1)$$

where A and g(t) are defined in Chapter 2, Equations (2.1b) and (2.1c).

As in Chapter 3, the only external field applied is $\delta B_z^e(r,t)$, and its maximum value is assumed to be one

percent of the equilibrium value B_o, which was previously determined in Chapter 2. The rest of the external fields are assumed equal to zero. Therefore, from Equation (2.1b),

$$g(t) = \begin{bmatrix} 0 \\ -W\, r\, (0.01)\, B_o \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Multiplying out and combining the terms described in Equation (4.1) results in the following nine partial differential equations:

$$(d/dt)\ \delta N(r,t) = -N^o\ \tilde{D}_r(\delta V_r(r,t)) \tag{4.2}$$

$$\begin{aligned}(d/dt)\ \delta V_r(r,t) = {} & a_{23}\ \delta V_\theta(r,t) + a_{24}\ r\ \delta\ V_z(r,t) \\ & + a_{25}\ \delta E_r(r,t) - a_{25}\ V_z{}^o\ \delta B_\theta(r,t) \\ & - a_{25}\ W\ r\ \delta B_z(r,t) + W\ r\ (.01)\ B_o\end{aligned} \tag{4.3}$$

$$(d/dt)\ \delta V_\theta(r,t) = -a_{23}\ \delta V_r(r,t) + a_{25}\ \delta E_\theta(r,t) \tag{4.4}$$

$$(d/dt)\ \delta V_z(r,t) = -a_{24}\ r\ \delta V_r(r,t) + a_{25}\ \delta E_z(r,t) \tag{4.5}$$

$$(d/dt)\ \delta E_e(r,t) = a_{52}\ \delta V_r(r,t) \tag{4.6}$$

$$(d/dt)\ \delta E_\theta(r,t) = \frac{-a_{52}\ W\ r\ \delta N(r,t)}{N^o} + a_{52}\ \delta V_\theta(r,t) - C^2\ D_r(\ \delta B_z(r,t)\ ) \tag{4.7}$$

$$(d/dt)\ \delta E_z(r,t) = \frac{a_{52}\ V_z^o}{N^o}\ \delta N(r,t) + a_{52}\ \delta V_z(r,t) + C^2\ \tilde{D}_r(\ \delta B_\theta(r,t)\ ) \tag{4.8}$$

$$(d/dt)\ \delta B_\theta(r,t) = D_r\ (\ \delta E_z(r,t)\ ) \tag{4.9}$$

$$(d/dt)\ \delta B_z(r,t) = -\ \tilde{D}_r(\ \delta E_\theta(r,t)\ ) \tag{4.10}$$

4-3 The Numerical Approximation

Equations (4.2) through (4.10) are the linear, time invariant, differential equations that govern the dynamic behavior of the particle beam. Of these nine equations, Equations (4.2), (4.7), (4.8), (4.9), and (4.10) require finite difference approximations for partial derivitives. This finite difference approximation is as follows:

$$D_r = \frac{f(x,t) - f(x-h,t)}{h}$$

where h is the spacing between points. After implementing this finite difference scheme, the five equations discussed above result in the following:

$$(d/dt)\ \delta N(r,t) = \frac{-N^o}{r}\left[\delta V_r(r,t) + r\left[\frac{\delta V_r(r,t) - \delta V_r(r,t)}{h}\right]\right] \tag{4.11}$$

$$(d/dt)\ \delta E_\theta(r,t) = \frac{-a_{52}\ W\ r}{N^o}\ \delta N(r,t) + a_{52}\ \delta V_\theta(r,t) - C^2\ \frac{\delta B_z(r,t) - \delta B_z(r-h,t)}{h} \tag{4.12}$$

$$(d/dt)\ \delta E_z(r,t) = \frac{a_{52}\ V_z^o}{N^o}\ \delta N(r,t) + a_{52}\ \delta V_z(r,t) + \frac{C^2}{r}\ \delta B_\theta(r,t) + r\left[\frac{\delta B_\theta(r,t) - \delta B_\theta(r-h,\ t)}{h}\right] \tag{4.13}$$

$$(d/dt)\ \delta B_\theta(r,t) = \frac{\delta E_z(r,t) - \delta E_z(r-h,t)}{h} \tag{4.14}$$

$$(d/dt)\ \delta B_z(r,t) = \frac{-1}{r}\left[\delta E_\theta(r,t) + r\left[\frac{\delta E_\theta(r,t) - \delta E_\theta(r-h,t)}{h}\right]\right] \tag{4.15}$$

4-4 Final Analysis and Solution Technique

We now have at hand, and in the correct form, the nine simultaneous differential equations that we will solve. Since these equations are much more complicated than those solved in Chapter 3, a numerical integration routine was used. The results from this integration routine are tabulated numerically in Tables 21 and 22. Figure 20 shows how the $\delta N(r,t)$ diverges with time. Since all nine solutions diverge in a similar manner no other figures are presented.

CHAPTER 5

RESULTS AND CONCLUSIONS

5.1 Introduction.

This chapter contains a discussion of the results obtained from the previous four chapters. Based upon these results, conclusions are drawn and recommendations are presented.

5-2 Closed Form Solution.

5-2.1 Step Input.

The results from the analysis presented in Section 2-4 are tabulated in Tables 1 thru 4 and are shown graphically in Figures 2 thru 5. It is observed from Figure 2 that a stable solution for $\delta N(r,t)$ is realized with no significant damping apparent. Identical results are seen from the equations which provide the solutions to $\delta V_z(r,t)$ and $\delta E_r(r,t)$ (Figures 4 and 5). This is consistent with the form of the solutions found in Equation (2.11). As can be seen in Equation (2.11) the solutions $\delta N(r,t)$, $\delta V_z(r,t)$, and $\delta E_r(r,t)$ all contain the same trigonometric function, (1-cos

[Ωt]). The only difference in the solutions is the amplitude of the trigonometric term; therefore, similar results are expected. Figure 3 shows that the solution of $\delta V_r(r,t)$ is also stable with no damping apparent.

An important result of this section is that the closed form solutions, as determined in Reference [1], can be used to evaluate the solutions to the system in question, and that these results make physical sense.

5-2.2 Input Rapidly Increasing to Constant.

The results from the analysis presented in Section 2-5.1 are tabulated in Tables 5 thru 8, and are shown graphically in Figures 6 thru 9. It should be noted that this type of input (Figure 18) is similar to the step input of Section 2-4. These two types of inputs are unequal only over a relatively short interval of time, approximately from $t=0.0$ to $t=4.0\times10^{-9}$ seconds, for the number of significant digits used in this effort. Therefore, it makes physical sense that the results of Section 2-5.1 are similar to those of Section 2-4. The lag in the solution that is apparent when comparing the data from Section 2-4 to the data of Section 2-5.1 is conjectured to be due to a slight lag in the values of the input due to the exponential term. Also, Figures 6 thru 9 show that the solutions from this section are stable and exhibit no apparent damping.

5-2.3 Input Rapidly Increasing to Constant Followed by a Rapidly Decrease to Zero.

The results from the analysis presented in Section 2-5.2 are tabulated in Tables 9 thru 16 and are shown graphically in Figures 10 thru 13. These figures clearly show two distinct functions for the solutions, $\underline{X}(t)$. The first function in all four figures spans the time interval between t=0.0 and $t=4.5\times10^{-7}$ seconds. Over this interval, the solutions, $\underline{X}(t)$, are almost identical to those values found in Section 2-5.1. This can be seen by comparing the values in Tables 9,11,13, and 15 with their related values in Tables 5 thru 8. Actually, the two solutions for $\underline{X}(t)$ are exact in comparison between t=0.0 and $t=4.45\times10^{-7}$ seconds, with the only differences encountered between $t >$ 4.45×10^{-7} to $t=4.5\times10^{7}$ seconds. This can be explained by referring to Equation (2.17), which shows how the external axial magnetic input was defined. Since, the input is exactly the same as that used for Section 2-5.1 over the interval $0 < t < t_1$, the results are expected to be identical. Over the interval $t_1 < t < t_2$ the input used is the cubic function defined in Equation (2.17b). The solutions are found by summing the results of Equation (2.6) integrated from t=0.0 to $t=t_1$, where the input is defined by Equation (2.17a), with the results of Equation (2.6) integrated from $t=t_1$ to $t \leq t_2$, where the input is defined by

integrated from $t=t_1$ to $t \le t_2$, where the input is defined by Equation (2.17b). Therefore, the results over this interval are expected to be slightly different. The second function in all four figures spans the interval of $t > 4.5 \times 10^{-7}$ to $t=\infty$ seconds. The input used over this interval is equal to zero; therefore the solutions are the summation of the previous works integrated over their associated intervals. Both these functions encountered on Figures 10 thru 13 are stable and exhibit no apparent damping.

5-3 Numerical Solution Technique.

The results from the analysis presented in Chapter 3 are tabulated in Tables 17 thru 20 and are shown graphically in Figures 14 thru 17. From these figures it is concluded that the solutions, X(t), are stable and show no apparent damping. It can also be concluded by comparing the data in Tables 17 thru 20 with their related data from Section 2-4, that the closed form solution techniques derived in Reference [1] produces excellent results. This conclusion is verified by the results of Chapter 3. It is conjectured that the small differences found between the closed form solution technique of Section 2-4 and the numerical solution technique of Chapter 3 are due to errors associated with the Chapter 3 method. Under this assumption, if the number of points across the radius of the beam were increased in order

to increase the accuracy of this solution technique, the results of this would more closely approximate those of Section 2-4.

5-4 Numerical Solution Technique Without Electrostatic Approximation

The results of the Chapter 4 analysis is presented in Tables 21 and 22. The solution for $\delta N(r,t)$ is shown graphically in Figure 20. It is observed from Figure 20 and from the data presented in Tables 21 and 22 that the solutions, $\underline{X}(t)$, are not stable for all time t. However, the data in Tables 21 and 22 do approximately agree with that of Section 2-4 over a interval of time, t=0.0 to $t<3.0\times10^{-11}$ second. Over this interval it can be concluded, due to the agreement in associated values, that the Electrostatic Approximation is an adequate one. For times other than that described above, the solution, $\underline{X}(t)$, diverges to $\pm\infty$. It is conjectured that this instability is being caused by any of the following anomalies. First, a numerical instability could have been encountered by the integration routine used to solve the differential equations. Secondly, the process of linearizing the differential equations might have eliminated a damping/restoring term present in the nonlinear equations. This term would had to have been in either $\delta E_\theta(r,t)$,

$\delta E_z(r,t)$, $\delta B_r(r,t)$, $\delta B_\theta(r,t)$, or $\delta B_z(r,t)$, since the Section 2-4 material was not effected in a similar manner. However, it is more likely that in choosing B_o such that we have the the minimun external magnetic field required to maintain rigid rotor equilibrium, we produce a system which borders on instability. Therefore, an increase in B_o would tend to stabilize this system.

5-5 Recommendations

The following recommendations are made as a result of this effort.

1. Investigate the divergence of the Chapter 4 study.
   - i. reexamine the derivation of the dynamic model found in Reference [1] paying close attention to the last four terms.
   - ii. continue work of chapter 4 by increasing B_o and recalculating the results. Should result in stable solutions.
2. Investigate a closed form solution to the dynamic system found in Reference [1] proir to the implementation of the electrostatic approximation.

APPENDIX A

TABLE 1
PERTURBED NUMBER DENSITY, SECTION 2-4

T1	T2	T3	T4	T5	R
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	5.00000E-04
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	1.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	1.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	2.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	2.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	3.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	3.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	4.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	4.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	5.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	5.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	6.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	6.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	7.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	7.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	8.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	8.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	9.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87119D+12	2.50009D+12	9.50000E-03

TABLE 2
PERTURBED RADIAL VELOCITY, SECTION 2-4

T1	T2	T3	T4	T5	R
-1.38946D+02	-4.50224D+01	1.24358D+02	8.53178D+01	-9.67127D+01	5.00000E-04
-2.77893D+02	-9.00448D+01	2.48716D+02	1.70636D+02	-1.93425D+02	1.00000E-03
-4.16839D+02	-1.35067D+02	3.73074D+02	2.55953D+02	-2.90138D+02	1.50000E-03
-5.55786D+02	-1.80090D+02	4.97432D+02	3.41271D+02	-3.86851D+02	2.00000E-03
-6.94732D+02	-2.25112D+02	6.21790D+02	4.26589D+02	-4.83564D+02	2.50000E-03
-8.33679D+02	-2.70134D+02	7.46148D+02	5.11907D+02	-5.80276D+02	3.00000E-03
-9.72625D+02	-3.15157D+02	8.70506D+02	5.97224D+02	-6.76989D+02	3.50000E-03
-1.11157D+03	-3.60179D+02	9.94864D+02	6.82542D+02	-7.73702D+02	4.00000E-03
-1.25052D+03	-4.05202D+02	1.11922D+03	7.67860D+02	-8.70415D+02	4.50000E-03
-1.38946D+03	-4.50224D+02	1.24358D+03	8.53178D+02	-9.67127D+02	5.00000E-03
-1.52841D+03	-4.95247D+02	1.36794D+03	9.38495D+02	-1.06384D+03	5.50000E-03
-1.66736D+03	-5.40269D+02	1.49230D+03	1.02381D+03	-1.16055D+03	6.00000E-03
-1.80630D+03	-5.85291D+02	1.61665D+03	1.10913D+03	-1.25727D+03	6.50000E-03
-1.94525D+03	-6.30314D+02	1.74101D+03	1.19445D+03	-1.35398D+03	7.00000E-03
-2.08420D+03	-6.75336D+02	1.86537D+03	1.27977D+03	-1.45069D+03	7.50000E-03
-2.22314D+03	-7.20359D+02	1.98973D+03	1.36508D+03	-1.54740D+03	8.00000E-03
-2.36209D+03	-7.65381D+02	2.11409D+03	1.45040D+03	-1.64412D+03	8.50000E-03
-2.50104D+03	-8.10403D+02	2.23844D+03	1.53572D+03	-1.74083D+03	9.00000E-03
-2.63998D+03	-8.55426D+02	2.36280D+03	1.62104D+03	-1.83754D+03	9.50000E-03

TABLE 3
PERTURBED AXIAL VELOCITY, SECTION 2-4

T1	T2	T3	T4	T5	R
-4.01335D-04	-9.32714D-04	-7.03560D-04	-9.79289D-05	-1.30842D-04	5.00000E-04
-1.60534D-03	-3.73086D-03	-2.81424D-03	-3.91716D-04	-5.23369D-04	1.00000E-03
-3.61202D-03	-8.39443D-03	-6.33204D-03	-8.81360D-04	-1.17758D-03	1.50000E-03
-6.42137D-03	-1.49234D-02	-1.12570D-02	-1.56686D-03	-2.09348D-03	2.00000E-03
-1.00334D-02	-2.33179D-02	-1.75890D-02	-2.44822D-03	-3.27106D-03	2.50000E-03
-1.44481D-02	-3.35777D-02	-2.53282D-02	-3.52544D-03	-4.71032D-03	3.00000E-03
-1.96654D-02	-4.57030D-02	-3.44744D-02	-4.79852D-03	-6.41127D-03	3.50000E-03
-2.56855D-02	-5.96937D-02	-4.50278D-02	-6.26745D-03	-8.37391D-03	4.00000E-03
-3.25082D-02	-7.55499D-02	-5.69884D-02	-7.93224D-03	-1.05982D-02	4.50000E-03
-4.01335D-02	-9.32714D-02	-7.03560D-02	-9.79289D-03	-1.30842D-02	5.00000E-03
-4.85616D-02	-1.12858D-01	-8.51308D-02	-1.18494D-02	-1.58319D-02	5.50000E-03
-5.77923D-02	-1.34311D-01	-1.01313D-01	-1.41018D-02	-1.88413D-02	6.00000E-03
-6.78257D-02	-1.57629D-01	-1.18902D-01	-1.65500D-02	-2.21124D-02	6.50000E-03
-7.86617D-02	-1.82812D-01	-1.37898D-01	-1.91941D-02	-2.56451D-02	7.00000E-03
-9.03005D-02	-2.09861D-0[illegible]	[illegible].58301D-01	-2.20340D-02	-2.94395D-02	7.50000E-03
-1.02742D-01	-2.38775D-01	-1.80111D-01	-2.50698D-02	-3.34956D-02	8.00000E-03
-1.15986D-01	-2.69554D-01	-2.03329D-01	-2.83015D-02	-3.78134D-02	8.50000E-03
-1.30033D-01	-3.02199D-01	-2.27953D-01	-3.17290D-02	-4.23929D-02	9.00000E-03
-1.44882D-01	-3.36710D-01	-2.53985D-01	-3.53523D-02	-4.72341D-02	9.50000E-03

TABLE 4
PERTURBED RADIAL ELECTRIC FIELD, SECTION 2-4

T1	T2	T3	T4	T5	R
3.46907D+01	8.06222D+01	6.08145D+01	8.46481D+00	1.13098D+01	5.00000E-04
6.93815D+01	1.61244D+02	1.21629D+02	1.69296D+01	2.26196D+01	1.00000E-03
1.04072D+02	2.41867D+02	1.82443D+02	2.53944D+01	3.39293D+01	1.50000E-03
1.38763D+02	3.22489D+02	2.43258D+02	3.38592D+01	4.52391D+01	2.00000E-03
1.73454D+02	4.03111D+02	3.04072D+02	4.23240D+01	5.65489D+01	2.50000E-03
2.08144D+02	4.83733D+02	3.64887D+02	5.07888D+01	6.78587D+01	3.00000E-03
2.42835D+02	5.64355D+02	4.25701D+02	5.92536D+01	7.91685D+01	3.50000E-03
2.77526D+02	6.44978D+02	4.86516D+02	6.77185D+01	9.04783D+01	4.00000E-03
3.12217D+02	7.25600D+02	5.47330D+02	7.61833D+01	1.01788D+02	4.50000E-03
3.46907D+02	8.06222D+02	6.08145D+02	8.46481D+01	1.13098D+02	5.00000E-03
3.81598D+02	8.86844D+02	6.68959D+02	9.31129D+01	1.24408D+02	5.50000E-03
4.16289D+02	9.67467D+02	7.29774D+02	1.01578D+02	1.35717D+02	6.00000E-03
4.50980D+02	1.04809D+03	7.90588D+02	1.10042D+02	1.47027D+02	6.50000E-03
4.85670D+02	1.12871D+03	8.51403D+02	1.18507D+02	1.58337D+02	7.00000E-03
5.20361D+02	1.20933D+03	9.12217D+02	1.26972D+02	1.69647D+02	7.50000E-03
5.55052D+02	1.28996D+03	9.73032D+02	1.35437D+02	1.80957D+02	8.00000E-03
5.89742D+02	1.37058D+03	1.03385D+03	1.43902D+02	1.92266D+02	8.50000E-03
6.24433D+02	1.45120D+03	1.09466D+03	1.52367D+02	2.03576D+02	9.00000E-03
6.59124D+02	1.53182D+03	1.15548D+03	1.60831D+02	2.14886D+02	9.50000E-03

TABLE 5
PERTURBED NUMBER DENSITY, SECTION 2-5.1

T1	T2	T3	T4	T5	R
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	5.00000E-04
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	1.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	1.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	2.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	2.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	3.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	3.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	4.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	4.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	5.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	5.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	6.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	6.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	7.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	7.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	8.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	8.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	9.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	9.50000E-03

TABLE 6
PERTURBED RADIAL VELOCITY, SECTION 2-5.1

T1	T2	T3	T4	T5	R
-1.38193D+02	-4.87404D+01	1.22400D+02	8.84013D+01	-9.37555D+01	5.00000E-04
-2.76386D+02	-9.74809D+01	2.44800D+02	1.76803D+02	-1.87511D+02	1.00000E-03
-4.14579D+02	-1.46221D+02	3.67200D+02	2.65204D+02	-2.81267D+02	1.50000E-03
-5.52773D+02	-1.94962D+02	4.89600D+02	3.53605D+02	-3.75022D+02	2.00000E-03
-6.90966D+02	-2.43702D+02	6.12000D+02	4.42007D+02	-4.68778D+02	2.50000E-03
-8.29159D+02	-2.92443D+02	7.34400D+02	5.30408D+02	-5.62533D+02	3.00000E-03
-9.67352D+02	-3.41183D+02	8.56800D+02	6.18809D+02	-6.56289D+02	3.50000E-03
-1.10555D+03	-3.89924D+02	9.79200D+02	7.07211D+02	-7.50044D+02	4.00000E-03
-1.24374D+03	-4.38664D+02	1.10160D+03	7.95612D+02	-8.43800D+02	4.50000E-03
-1.38193D+03	-4.87404D+02	1.22400D+03	8.84013D+02	-9.37555D+02	5.00000E-03
-1.52013D+03	-5.36145D+02	1.34640D+03	9.72415D+02	-1.03131D+03	5.50000E-03
-1.65832D+03	-5.84885D+02	1.46880D+03	1.06082D+03	-1.12507D+03	6.00000E-03
-1.79651D+03	-6.33626D+02	1.59120D+03	1.14922D+03	-1.21882D+03	6.50000E-03
-1.93470D+03	-6.82366D+02	1.71360D+03	1.23762D+03	-1.31258D+03	7.00000E-03
-2.07290D+03	-7.31107D+02	1.83600D+03	1.32602D+03	-1.40633D+03	7.50000E-03
-2.21109D+03	-7.79847D+02	1.95840D+03	1.41442D+03	-1.50009D+03	8.00000E-03
-2.34928D+03	-8.28588D+02	2.08080D+03	1.50282D+03	-1.59384D+03	8.50000E-03
-2.48748D+03	-8.77328D+02	2.20320D+03	1.59122D+03	-1.68760D+03	9.00000E-03
-2.62567D+03	-9.26068D+02	2.32560D+03	1.67963D+03	-1.78135D+03	9.50000E-03

TABLE 7
PERTURBED AXIAL VELOCITY, SECTION 2-5.1

T1	T2	T3	T4	T5	R
-3.87972D-04	-9.28756D-04	-7.15633D-04	-1.05792D-04	-1.21309D-04	5.00000E-04
-1.55189D-03	-3.71502D-03	-2.86253D-03	-4.23167D-04	-4.85236D-04	1.00000E-03
-3.49175D-03	-8.35880D-03	-6.44070D-03	-9.52126D-04	-1.09178D-03	1.50000E-03
-6.20755D-03	-1.48601D-02	-1.14501D-02	-1.69267D-03	-1.94094D-03	2.00000E-03
-9.69929D-03	-2.32189D-02	-1.78908D-02	-2.64479D-03	-3.03273D-03	2.50000E-03
-1.39670D-02	-3.34352D-02	-2.57628D-02	-3.80850D-03	-4.36712D-03	3.00000E-03
-1.90106D-02	-4.55091D-02	-3.50660D-02	-5.18380D-03	-5.94414D-03	3.50000E-03
-2.48302D-02	-5.94404D-02	-4.58005D-02	-6.77067D-03	-7.76378D-03	4.00000E-03
-3.14257D-02	-7.52293D-02	-5.79663D-02	-8.56913D-03	-9.82603D-03	4.50000E-03
-3.87972D-02	-9.28756D-02	-7.15634D-02	-1.05792D-02	-1.21309D-02	5.00000E-03
-4.69446D-02	-1.12380D-01	-8.65917D-02	-1.28008D-02	-1.46784D-02	5.50000E-03
-5.58679D-02	-1.33741D-01	-1.03051D-01	-1.52340D-02	-1.74685D-02	6.00000E-03
-6.55672D-02	-1.56960D-01	-1.20942D-01	-1.78788D-02	-2.05012D-02	6.50000E-03
-7.60425D-02	-1.82036D-01	-1.40264D-01	-2.07352D-02	-2.37766D-02	7.00000E-03
-8.72937D-02	-2.08970D-01	-1.61018D-01	-2.38031D-02	-2.72945D-02	7.50000E-03
-9.93208D-02	-2.37762D-01	-1.83202D-01	-2.70827D-02	-3.10551D-02	8.00000E-03
-1.12124D-01	-2.68411D-01	-2.06818D-01	-3.05738D-02	-3.50583D-02	8.50000E-03
-1.25703D-01	-3.00917D-01	-2.31865D-01	-3.42765D-02	-3.93041D-02	9.00000E-03
-1.40058D-01	-3.35281D-01	-2.58344D-01	-3.81908D-02	-4.37926D-02	9.50000E-03

TABLE 8
PERTURBED RADIAL ELECTRIC FIELD, SECTION 2-5.1

T1	T2	T3	T4	T5	R
3.35356D+01	8.02801D+01	6.18581D+01	9.14445D+00	1.04857D+01	5.00000E-04
6.70712D+01	1.60560D+02	1.23716D+02	1.82889D+01	2.09715D+01	1.00000E-03
1.00607D+02	2.40840D+02	1.85574D+02	2.74334D+01	3.14572D+01	1.50000E-03
1.34142D+02	3.21120D+02	2.47432D+02	3.65778D+01	4.19430D+01	2.00000E-03
1.67678D+02	4.01400D+02	3.09291D+02	4.57223D+01	5.24287D+01	2.50000E-03
2.01214D+02	4.81680D+02	3.71149D+02	5.48667D+01	6.29144D+01	3.00000E-03
2.34749D+02	5.61960D+02	4.33007D+02	6.40112D+01	7.34002D+01	3.50000E-03
2.68285D+02	6.42241D+02	4.94865D+02	7.31556D+01	8.38859D+01	4.00000E-03
3.01820D+02	7.22521D+02	5.56723D+02	8.23001D+01	9.43717D+01	4.50000E-03
3.35356D+02	8.02801D+02	6.18581D+02	9.14446D+01	1.04857D+02	5.00000E-03
3.68892D+02	8.83081D+02	6.80439D+02	1.00589D+02	1.15343D+02	5.50000E-03
4.02427D+02	9.63361D+02	7.42297D+02	1.09733D+02	1.25829D+02	6.00000E-03
4.35963D+02	1.04364D+03	8.04156D+02	1.18878D+02	1.36315D+02	6.50000E-03
4.69499D+02	1.12392D+03	8.66014D+02	1.28022D+02	1.46800D+02	7.00000E-03
5.03034D+02	1.20420D+03	9.27872D+02	1.37167D+02	1.57286D+02	7.50000E-03
5.36570D+02	1.28448D+03	9.89730D+02	1.46311D+02	1.67772D+02	8.00000E-03
5.70105D+02	1.36476D+03	1.05159D+03	1.55456D+02	1.78258D+02	8.50000E-03
6.03641D+02	1.44504D+03	1.11345D+03	1.64600D+02	1.88743D+02	9.00000E-03
6.37177D+02	1.52532D+03	1.17530D+03	1.73745D+02	1.99229D+02	9.50000E-03

TABLE 9

PERTURBED NUMBER DENSITY, SECTION 2-5.2

T1	T2	T3	T4	T5	R
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	5.00000E-04
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	1.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	1.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	2.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	2.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	3.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	3.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	4.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	4.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	5.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	5.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	6.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	6.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	7.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	7.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	8.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	8.50000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	9.00000E-03
7.41324D+12	1.77464D+13	1.36741D+13	2.02143D+12	2.31793D+12	9.50000E-03

TABLE 10

PERTURBED NUMBER DENSITY, SECTION 2-5.2

T6	T7	T8	T9	T10	R
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	5.00000E-04
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	1.00000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	1.50000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	2.00000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	2.50000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	3.00000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	3.50000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	4.00000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	4.50000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	5.00000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	5.50000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	6.00000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	6.50000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	7.00000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	7.50000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	8.00000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	8.50000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	9.00000E-03
-1.37514D+10	2.86405D+09	1.46796D+10	1.89266D+09	-1.40664D+10	9.50000E-03

TABLE 11

PERTURBED RADIAL VELOCITY, SECTION 2-5.2

T1	T2	T3	T4	T5	R
-1.38193D+02	-4.87404D+01	1.22400D+02	8.84013D+01	-9.37555D+01	5.00000E-04
-2.76386D+02	-9.74809D+01	2.44800D+02	1.76803D+02	-1.87511D+02	1.00000E-03
-4.14579D+02	-1.46221D+02	3.67200D+02	2.65204D+02	-2.81267D+02	1.50000E-03
-5.52773D+02	-1.94962D+02	4.89600D+02	3.53605D+02	-3.75022D+02	2.00000E-03
-6.90966D+02	-2.43702D+02	6.12000D+02	4.42007D+02	-4.68778D+02	2.50000E-03
-8.29159D+02	-2.92443D+02	7.34400D+02	5.30408D+02	-5.62533D+02	3.00000E-03
-9.67352D+02	-3.41183D+02	8.56800D+02	6.18809D+02	-6.56289D+02	3.50000E-03
-1.10555D+03	-3.89924D+02	9.79200D+02	7.07211D+02	-7.50044D+02	4.00000E-03
-1.24374D+03	-4.38664D+02	1.10160D+03	7.95612D+02	-8.43800D+02	4.50000E-03
-1.38193D+03	-4.87404D+02	1.22400D+03	8.84013D+02	-9.37555D+02	5.00000E-03
-1.52013D+03	-5.36145D+02	1.34640D+03	9.72415D+02	-1.03131D+03	5.50000E-03
-1.65832D+03	-5.84885D+02	1.46880D+03	1.06082D+03	-1.12507D+03	6.00000E-03
-1.79651D+03	-6.33626D+02	1.59120D+03	1.14922D+03	-1.21882D+03	6.50000E-03
-1.93470D+03	-6.82366D+02	1.71360D+03	1.23762D+03	-1.31258D+03	7.00000E-03
-2.07290D+03	-7.31107D+02	1.83600D+03	1.32602D+03	-1.40633D+03	7.50000E-03
-2.21109D+03	-7.79847D+02	1.95840D+03	1.41442D+03	-1.50009D+03	8.00000E-03
-2.34928D+03	-8.28588D+02	2.08080D+03	1.50282D+03	-1.59384D+03	8.50000E-03
-2.48748D+03	-8.77328D+02	2.20320D+03	1.59122D+03	-1.68760D+03	9.00000E-03
-2.62567D+03	-9.26068D+02	2.32560D+03	1.67963D+03	-1.78135D+03	9.50000E-03

TABLE 12

PERTURBED RADIAL VELOCITY, SECTION 2-5.2

T6	T7	T8	T9	T10	R
5.26128D-01	-8.61391D+00	-3.31727D+00	7.53902D+00	5.76013D+00	5.00000E-04
1.05226D+00	-1.72278D+01	-6.63454D+00	1.50780D+01	1.15203D+01	1.00000E-03
1.57838D+00	-2.58417D+01	-9.95182D+00	2.26171D+01	1.72804D+01	1.50000E-03
2.10451D+00	-3.44556D+01	-1.32691D+01	3.01561D+01	2.30405D+01	2.00000E-03
2.63064D+00	-4.30695D+01	-1.65864D+01	3.76951D+01	2.88006D+01	2.50000E-03
3.15677D+00	-5.16835D+01	-1.99036D+01	4.52341D+01	3.45608D+01	3.00000E-03
3.68290D+00	-6.02974D+01	-2.32209D+01	5.27731D+01	4.03209D+01	3.50000E-03
4.20903D+00	-6.89113D+01	-2.65382D+01	6.03121D+01	4.60810D+01	4.00000E-03
4.73515D+00	-7.75252D+01	-2.98555D+01	6.78512D+01	5.18412D+01	4.50000E-03
5.26128D+00	-8.61391D+01	-3.31727D+01	7.53902D+01	5.76013D+01	5.00000E-03
5.78741D+00	-9.47530D+01	-3.64900D+01	8.29292D+01	6.33614D+01	5.50000E-03
6.31354D+00	-1.03367D+02	-3.98073D+01	9.04682D+01	6.91216D+01	6.00000E-03
6.83967D+00	-1.11981D+02	-4.31245D+01	9.80072D+01	7.48817D+01	6.50000E-03
7.36580D+00	-1.20595D+02	-4.64418D+01	1.05546D+02	8.06418D+01	7.00000E-03
7.89192D+00	-1.29209D+02	-4.97591D+01	1.13085D+02	8.64019D+01	7.50000E-03
8.41805D+00	-1.37823D+02	-5.30764D+01	1.20624D+02	9.21621D+01	8.00000E-03
8.94418D+00	-1.46436D+02	-5.63936D+01	1.28163D+02	9.79222D+01	8.50000E-03
9.47031D+00	-1.55050D+02	-5.97109D+01	1.35702D+02	1.03682D+02	9.00000E-03
9.99644D+00	-1.63664D+02	-6.30282D+01	1.43241D+02	1.09442D+02	9.50000E-03

TABLE 13

PERTURBED AXIAL VELOCITY, SECTION 2-5.2

T1	T2	T3	T4	T5	R
-3.87972D-04	-9.28756D-04	-7.15633D-04	-1.05792D-04	-1.21309D-04	5.00000E-04
-1.55189D-03	-3.71502D-03	-2.86253D-03	-4.23167D-04	-4.85236D-04	1.00000E-03
-3.49175D-03	-8.35880D-03	-6.44070D-03	-9.52126D-04	-1.09178D-03	1.50000E-03
-6.20755D-03	-1.48601D-02	-1.14501D-02	-1.69267D-03	-1.94094D-03	2.00000E-03
-9.69929D-03	-2.32189D-02	-1.78908D-02	-2.64479D-03	-3.03273D-03	2.50000E-03
-1.39670D-02	-3.34352D-02	-2.57628D-02	-3.80850D-03	-4.36712D-03	3.00000E-03
-1.90106D-02	-4.55091D-02	-3.50660D-02	-5.18380D-03	-5.94414D-03	3.50000E-03
-2.48302D-02	-5.94404D-02	-4.58005D-02	-6.77067D-03	-7.76378D-03	4.00000E-03
-3.14257D-02	-7.52293D-02	-5.79663D-02	-8.56913D-03	-9.82603D-03	4.50000E-03
-3.87972D-02	-9.28756D-02	-7.15634D-02	-1.05792D-02	-1.21309D-02	5.00000E-03
-4.69446D-02	-1.12380D-01	-8.65917D-02	-1.28008D-02	-1.46784D-02	5.50000E-03
-5.58679D-02	-1.33741D-01	-1.03051D-01	-1.52340D-02	-1.74685D-02	6.00000E-03
-6.55672D-02	-1.56960D-01	-1.20942D-01	-1.78788D-02	-2.05012D-02	6.50000E-03
-7.60425D-02	-1.82036D-01	-1.40264D-01	-2.07352D-02	-2.37766D-02	7.00000E-03
-8.72937D-02	-2.08970D-01	-1.61018D-01	-2.38031D-02	-2.72945D-02	7.50000E-03
-9.93208D-02	-2.37762D-01	-1.83202D-01	-2.70827D-02	-3.10551D-02	8.00000E-03
-1.12124D-01	-2.68411D-01	-2.06818D-01	-3.05738D-02	-3.50583D-02	8.50000E-03
-1.25703D-01	-3.00917D-01	-2.31865D-01	-3.42765D-02	-3.93041D-02	9.00000E-03
-1.40058D-01	-3.35281D-01	-2.58344D-01	-3.81908D-02	-4.37926D-02	9.50000E-03

TABLE 14

PERTURBED AXIAL VELOCITY, SECTION 2-5.2

T6	T7	T8	T9	T10	R
7.19677D-07	-1.49890D-07	-7.68256D-07	-9.90526D-08	7.36163D-07	5.00000E-04
2.87871D-06	-5.99561D-07	-3.07302D-06	-3.96210D-07	2.94465D-06	1.00000E-03
6.47710D-06	-1.34901D-06	-6.91430D-06	-8.91473D-07	6.62547D-06	1.50000E-03
1.15148D-05	-2.39824D-06	-1.22921D-05	-1.58484D-06	1.17786D-05	2.00000E-03
1.79919D-05	-3.74726D-06	-1.92064D-05	-2.47631D-06	1.84041D-05	2.50000E-03
2.59084D-05	-5.39605D-06	-2.76572D-05	-3.56589D-06	2.65019D-05	3.00000E-03
3.52642D-05	-7.34463D-06	-3.76445D-05	-4.85358D-06	3.60720D-05	3.50000E-03
4.60594D-05	-9.59298D-06	-4.91684D-05	-6.33937D-06	4.71145D-05	4.00000E-03
5.82939D-05	-1.21411D-05	-6.22287D-05	-8.02326D-06	5.96292D-05	4.50000E-03
7.19677D-05	-1.49890D-05	-7.68256D-05	-9.90526D-06	7.36163D-05	5.00000E-03
8.70810D-05	-1.81367D-05	-9.29590D-05	-1.19854D-05	8.90758D-05	5.50000E-03
1.03634D-04	-2.15842D-05	-1.10629D-04	-1.42636D-05	1.06008D-04	6.00000E-03
1.21626D-04	-2.53315D-05	-1.29835D-04	-1.67399D-05	1.24412D-04	6.50000E-03
1.41057D-04	-2.93785D-05	-1.50578D-04	-1.94143D-05	1.44288D-04	7.00000E-03
1.61927D-04	-3.37253D-05	-1.72858D-04	-2.22868D-05	1.65637D-04	7.50000E-03
1.84237D-04	-3.83719D-05	-1.96674D-04	-2.53575D-05	1.88458D-04	8.00000E-03
2.07987D-04	-4.33183D-05	-2.22026D-04	-2.86262D-05	2.12751D-04	8.50000E-03
2.33175D-04	-4.85645D-05	-2.48915D-04	-3.20930D-05	2.38517D-04	9.00000E-03
2.59804D-04	-5.41104D-05	-2.77340D-04	-3.57580D-05	2.65755D-04	9.50000E-03

TABLE 15

PERTURBED RADIAL ELECTRIC FIELD, SECTION 2-5.2

T1	T2	T3	T4	T5	R
3.35356D+01	8.02801D+01	6.18581D+01	9.14445D+00	1.04857D+01	5.00000E-04
6.70712D+01	1.60560D+02	1.23716D+02	1.82889D+01	2.09715D+01	1.00000E-03
1.00607D+02	2.40840D+02	1.85574D+02	2.74334D+01	3.14572D+01	1.50000E-03
1.34142D+02	3.21120D+02	2.47432D+02	3.65778D+01	4.19430D+01	2.00000E-03
1.67678D+02	4.01400D+02	3.09291D+02	4.57223D+01	5.24287D+01	2.50000E-03
2.01214D+02	4.81680D+02	3.71149D+02	5.48667D+01	6.29144D+01	3.00000E-03
2.34749D+02	5.61960D+02	4.33007D+02	6.40112D+01	7.34002D+01	3.50000E-03
2.68285D+02	6.42241D+02	4.94865D+02	7.31556D+01	8.38859D+01	4.00000E-03
3.01820D+02	7.22521D+02	5.56723D+02	8.23001D+01	9.43717D+01	4.50000E-03
3.35356D+02	8.02801D+02	6.18581D+02	9.14446D+01	1.04857D+02	5.00000E-03
3.68892D+02	8.83081D+02	6.80439D+02	1.00589D+02	1.15343D+02	5.50000E-03
4.02427D+02	9.63361D+02	7.42297D+02	1.09733D+02	1.25829D+02	6.00000E-03
4.35963D+02	1.04364D+03	8.04156D+02	1.18878D+02	1.36315D+02	6.50000E-03
4.69499D+02	1.12392D+03	8.66014D+02	1.28022D+02	1.46800D+02	7.00000E-03
5.03034D+02	1.20420D+03	9.27872D+02	1.37167D+02	1.57286D+02	7.50000E-03
5.36570D+02	1.28448D+03	9.89730D+02	1.46311D+02	1.67772D+02	8.00000E-03
5.70105D+02	1.36476D+03	1.05159D+03	1.55456D+02	1.78258D+02	8.50000E-03
6.03641D+02	1.44504D+03	1.11345D+03	1.64600D+02	1.88743D+02	9.00000E-03
6.37177D+02	1.52532D+03	1.17530D+03	1.73745D+02	1.99229D+02	9.50000E-03

TABLE 16

PERTURBED RADIAL ELECTRIC FIELD, SECTION 2-5.2

T6	T7	T8	T9	T10	R
-6.22077D-02	1.29563D-02	6.64067D-02	8.56193D-03	-6.36327D-02	5.00000E-04
-1.24415D-01	2.59125D-02	1.32813D-01	1.71239D-02	-1.27265D-01	1.00000E-03
-1.86623D-01	3.88688D-02	1.99220D-01	2.56858D-02	-1.90898D-01	1.50000E-03
-2.48831D-01	5.18250D-02	2.65627D-01	3.42477D-02	-2.54531D-01	2.00000E-03
-3.11038D-01	6.47813D-02	3.32034D-01	4.28097D-02	-3.18163D-01	2.50000E-03
-3.73246D-01	7.77375D-02	3.98440D-01	5.13716D-02	-3.81796D-01	3.00000E-03
-4.35454D-01	9.06938D-02	4.64847D-01	5.99335D-02	-4.45429D-01	3.50000E-03
-4.97661D-01	1.03650D-01	5.31254D-01	6.84955D-02	-5.09061D-01	4.00000E-03
-5.59869D-01	1.16606D-01	5.97660D-01	7.70574D-02	-5.72694D-01	4.50000E-03
-6.22077D-01	1.29563D-01	6.64067D-01	8.56193D-02	-6.36327D-01	5.00000E-03
-6.84284D-01	1.42519D-01	7.30474D-01	9.41813D-02	-6.99959D-01	5.50000E-03
-7.46492D-01	1.55475D-01	7.96881D-01	1.02743D-01	-7.63592D-01	6.00000E-03
-8.08700D-01	1.68431D-01	8.63287D-01	1.11305D-01	-8.27225D-01	6.50000E-03
-8.70907D-01	1.81388D-01	9.29694D-01	1.19867D-01	-8.90857D-01	7.00000E-03
-9.33115D-01	1.94344D-01	9.96101D-01	1.28429D-01	-9.54490D-01	7.50000E-03
-9.95323D-01	2.07300D-01	1.06251D+00	1.36991D-01	-1.01812D+00	8.00000E-03
-1.05753D+00	2.20256D-01	1.12891D+00	1.45553D-01	-1.08176D+00	8.50000E-03
-1.11974D+00	2.33213D-01	1.19532D+00	1.54115D-01	-1.14539D+00	9.00000E-03
-1.18195D+00	2.46169D-01	1.26173D+00	1.62677D-01	-1.20902D+00	9.50000E-03

TABLE 17
PERTURBED NUMBER DENSITY, CHAPTER 3

T1	T2	T3	T4	T5	R
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	5.00000E-04
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	1.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	1.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	2.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	2.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	3.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	3.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	4.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	4.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	5.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	5.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	6.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	6.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	7.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	7.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	8.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	8.50000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	9.00000E-03
7.66858D+12	1.78220D+13	1.34434D+13	1.87117D+12	2.50012D+12	9.50000E-03

TABLE 18
PERTURBED RADIAL VELOCITY, CHAPTER 3

T1	T2	T3	T4	T5	R
-1.38946D+02	-4.50221D+01	1.24358D+02	8.53172D+01	-9.67132D+01	5.00000E-04
-2.77893D+02	-9.00442D+01	2.48716D+02	1.70634D+02	-1.93426D+02	1.00000E-03
-4.16839D+02	-1.35066D+02	3.73074D+02	2.55952D+02	-2.90140D+02	1.50000E-03
-5.55786D+02	-1.80088D+02	4.97433D+02	3.41269D+02	-3.86853D+02	2.00000E-03
-6.94732D+02	-2.25110D+02	6.21791D+02	4.26586D+02	-4.83566D+02	2.50000E-03
-8.33679D+02	-2.70133D+02	7.46149D+02	5.11903D+02	-5.80279D+02	3.00000E-03
-9.72625D+02	-3.15155D+02	8.70507D+02	5.97221D+02	-6.76992D+02	3.50000E-03
-1.11157D+03	-3.60177D+02	9.94865D+02	6.82538D+02	-7.73706D+02	4.00000E-03
-1.25052D+03	-4.05199D+02	1.11922D+03	7.67855D+02	-8.70419D+02	4.50000E-03
-1.38946D+03	-4.50221D+02	1.24358D+03	8.53172D+02	-9.67132D+02	5.00000E-03
-1.52841D+03	-4.95243D+02	1.36794D+03	9.38490D+02	-1.06385D+03	5.50000E-03
-1.66736D+03	-5.40265D+02	1.49230D+03	1.02381D+03	-1.16056D+03	6.00000E-03
-1.80630D+03	-5.85287D+02	1.61666D+03	1.10912D+03	-1.25727D+03	6.50000E-03
-1.94525D+03	-6.30309D+02	1.74101D+03	1.19444D+03	-1.35398D+03	7.00000E-03
-2.08420D+03	-6.75332D+02	1.86537D+03	1.27976D+03	-1.45070D+03	7.50000E-03
-2.22314D+03	-7.20354D+02	1.98973D+03	1.36508D+03	-1.54741D+03	8.00000E-03
-2.36209D+03	-7.65376D+02	2.11409D+03	1.45039D+03	-1.64412D+03	8.50000E-03
-2.50104D+03	-8.10398D+02	2.23845D+03	1.53571D+03	-1.74084D+03	9.00000E-03
-2.63998D+03	-8.55420D+02	2.36280D+03	1.62103D+03	-1.83755D+03	9.50000E-03

TABLE 19
PERTURBED AXIAL VELOCITY, CHAPTER 3

T1	T2	T3	T4	T5	R
-4.01335D-04	-9.32713D-04	-7.03558D-04	-9.79276D-05	-1.30844D-04	5.00000E-04
-1.60534D-03	-3.73085D-03	-2.81423D-03	-3.91710D-04	-5.23375D-04	1.00000E-03
-3.61202D-03	-8.39442D-03	-6.33202D-03	-8.81348D-04	-1.17759D-03	1.50000E-03
-6.42136D-03	-1.49234D-02	-1.12569D-02	-1.56684D-03	-2.09350D-03	2.00000E-03
-1.00334D-02	-2.33178D-02	-1.75889D-02	-2.44819D-03	-3.27110D-03	2.50000E-03
-1.44481D-02	-3.35777D-02	-2.53281D-02	-3.52539D-03	-4.71038D-03	3.00000E-03
-1.96654D-02	-4.57030D-02	-3.44743D-02	-4.79845D-03	-6.41135D-03	3.50000E-03
-2.56855D-02	-5.96937D-02	-4.50277D-02	-6.26737D-03	-8.37401D-03	4.00000E-03
-3.25082D-02	-7.55498D-02	-5.69882D-02	-7.93213D-03	-1.05984D-02	4.50000E-03
-4.01335D-02	-9.32714D-02	-7.03558D-02	-9.79276D-03	-1.30844D-02	5.00000E-03
-4.85616D-02	-1.12858D-01	-8.51305D-02	-1.18492D-02	-1.58321D-02	5.50000E-03
-5.77923D-02	-1.34311D-01	-1.01312D-01	-1.41016D-02	-1.88415D-02	6.00000E-03
-6.78257D-02	-1.57629D-01	-1.18901D-01	-1.65498D-02	-2.21126D-02	6.50000E-03
-7.86617D-02	-1.82812D-01	-1.37897D-01	-1.91938D-02	-2.56454D-02	7.00000E-03
-9.03005D-02	-2.09861D-01	-1.58301D-01	-2.20337D-02	-2.94399D-02	7.50000E-03
-1.02742D-01	-2.38775D-01	-1.80111D-01	-2.50695D-02	-3.34960D-02	8.00000E-03
-1.15986D-01	-2.69554D-01	-2.03328D-01	-2.83011D-02	-3.78139D-02	8.50000E-03
-1.30033D-01	-3.02199D-01	-2.27953D-01	-3.17285D-02	-4.23934D-02	9.00000E-03
-1.44882D-01	-3.36710D-01	-2.53984D-01	-3.53519D-02	-4.72346D-02	9.50000E-03

TABLE 20
PERTURBED RADIAL ELECTRIC FIELD, CHAPTER 3

T1	T2	T3	T4	T5	R
3.46907D+01	8.06221D+01	6.08143D+01	8.46469D+00	1.13099D+01	5.00000E-04
6.93815D+01	1.61244D+02	1.21629D+02	1.69294D+01	2.26198D+01	1.00000E-03
1.04072D+02	2.41866D+02	1.82443D+02	2.53941D+01	3.39297D+01	1.50000E-03
1.38763D+02	3.22488D+02	2.43257D+02	3.38588D+01	4.52397D+01	2.00000E-03
1.73454D+02	4.03111D+02	3.04072D+02	4.23234D+01	5.65496D+01	2.50000E-03
2.08144D+02	4.83733D+02	3.64886D+02	5.07881D+01	6.78595D+01	3.00000E-03
2.42835D+02	5.64355D+02	4.25700D+02	5.92528D+01	7.91694D+01	3.50000E-03
2.77526D+02	6.44977D+02	4.86514D+02	6.77175D+01	9.04793D+01	4.00000E-03
3.12217D+02	7.25599D+02	5.47329D+02	7.61822D+01	1.01789D+02	4.50000E-03
3.46907D+02	8.06221D+02	6.08143D+02	8.46469D+01	1.13099D+02	5.00000E-03
3.81598D+02	8.86843D+02	6.68957D+02	9.31116D+01	1.24409D+02	5.50000E-03
4.16289D+02	9.67466D+02	7.29772D+02	1.01576D+02	1.35719D+02	6.00000E-03
4.50980D+02	1.04809D+03	7.90586D+02	1.10041D+02	1.47029D+02	6.50000E-03
4.85670D+02	1.12871D+03	8.51400D+02	1.18506D+02	1.58339D+02	7.00000E-03
5.20361D+02	1.20933D+03	9.12215D+02	1.26970D+02	1.69649D+02	7.50000E-03
5.55052D+02	1.28995D+03	9.73029D+02	1.35435D+02	1.80959D+02	8.00000E-03
5.89742D+02	1.37058D+03	1.03384D+03	1.43900D+02	1.92269D+02	8.50000E-03
6.24433D+02	1.45120D+03	1.09466D+03	1.52364D+02	2.03578D+02	9.00000E-03
6.59124D+02	1.53182D+03	1.15547D+03	1.60829D+02	2.14888D+02	9.50000E-03

TABLE 21
PERTURBED QUANTITIES, CHAPTER 4

$N(r,t)$	$V_r(r,t)$	$V_\theta(r,t)$	TIME
.470282725889E+05	-.142750907355E-01	.308797007098E-14	.360000000000E-11
.209595140067E+06	-.301363024211E-01	.159899574921E-12	.760000000000E-11
.488281191390E+06	-.459975113493E-01	.565086538576E-11	.116000000000E-10
.883086275831E+06	-.618586490873E-01	.169259682091E-09	.156000000000E-10
.139400601167E+07	-.777176876989E-01	.503118902514E-08	.196000000000E-10
.202091031538E+07	-.935143633136E-01	.149524674249E-06	.236000000000E-10
.275993349555E+07	-.107457736718E+00	.444364747439E-05	.276000000000E-10
.349619332570E+07	-.663238334625E-01	.132058412945E-03	.316000000000E-10
.815564535957E+06	.161162313860E+01	.392457559581E-02	.356000000000E-10
-.106744536042E+09	.519331527439E+02	.116632391137E+00	.396000000000E-10

$V_z(r,t)$	$E_r(r,t)$	$E_\theta(r,t)$	TIME
-.159276825847E-10	.212743966624E-06	.329232209408E-10	.360000000000E-11
-.708281915914E-09	.948155205973E-06	.144835522286E-08	.760000000000E-11
-.246686497795E-07	.220886015509E-05	.501085863021E-07	.116000000000E-10
-.738177854506E-06	.399485813213E-05	.149851789534E-05	.156000000000E-10
-.219407959395E-04	.630612931530E-05	.445384221883E-04	.196000000000E-10
-.652068521520E-03	.914208523973E-05	.132365445078E-02	.236000000000E-10
-.193784885670E-01	.124852384989E-04	.393369999991E-01	.276000000000E-10
-.575899065330E+00	.158158910640E-04	.116903551683E+01	.316000000000E-10
-.171148461225E+02	.368940692197E-05	.347419611898E+02	.356000000000E-10
-.508627080421E+03	-.482885182955E-03	.103247801128E+04	.396000000000E-10

TABLE 22
PERTURBED QUANTITIES, CHAPTER 4

$E_z(r,t)$	$B_0(r,t)$	$B_z(r,t)$	TIME
-.143576253407E-06	-.281178995059E-15	-.128953332582E-18	.360000000000E-11
-.631619296526E-05	-.145598567192E-13	-.667739083949E-17	.760000000000E-11
-.218520633133E-03	-.514546648399E-12	-.235979593946E-15	.116000000000E-10
-.653494946509E-02	-.154121530392E-10	-.706826801289E-14	.156000000000E-10
-.194229471106E+00	-.458121238718E-09	-.210101968844E-12	.196000000000E-10
-.577238014437E+01	-.136151571017E-07	-.624413598721E-11	.236000000000E-10
-.171546371185E+03	-.404621904528E-06	-.185566290305E-09	.276000000000E-10
-.509809595811E+04	-.120247447311E-04	-.551474659836E-08	.316000000000E-10
-.151507673948E+06	-.357357162375E-03	-.163889898677E-06	.356000000000E-10
-.450257661152E+07	-.106201089316E-01	-.487055741426E-05	.396000000000E-10

APPENDIX B

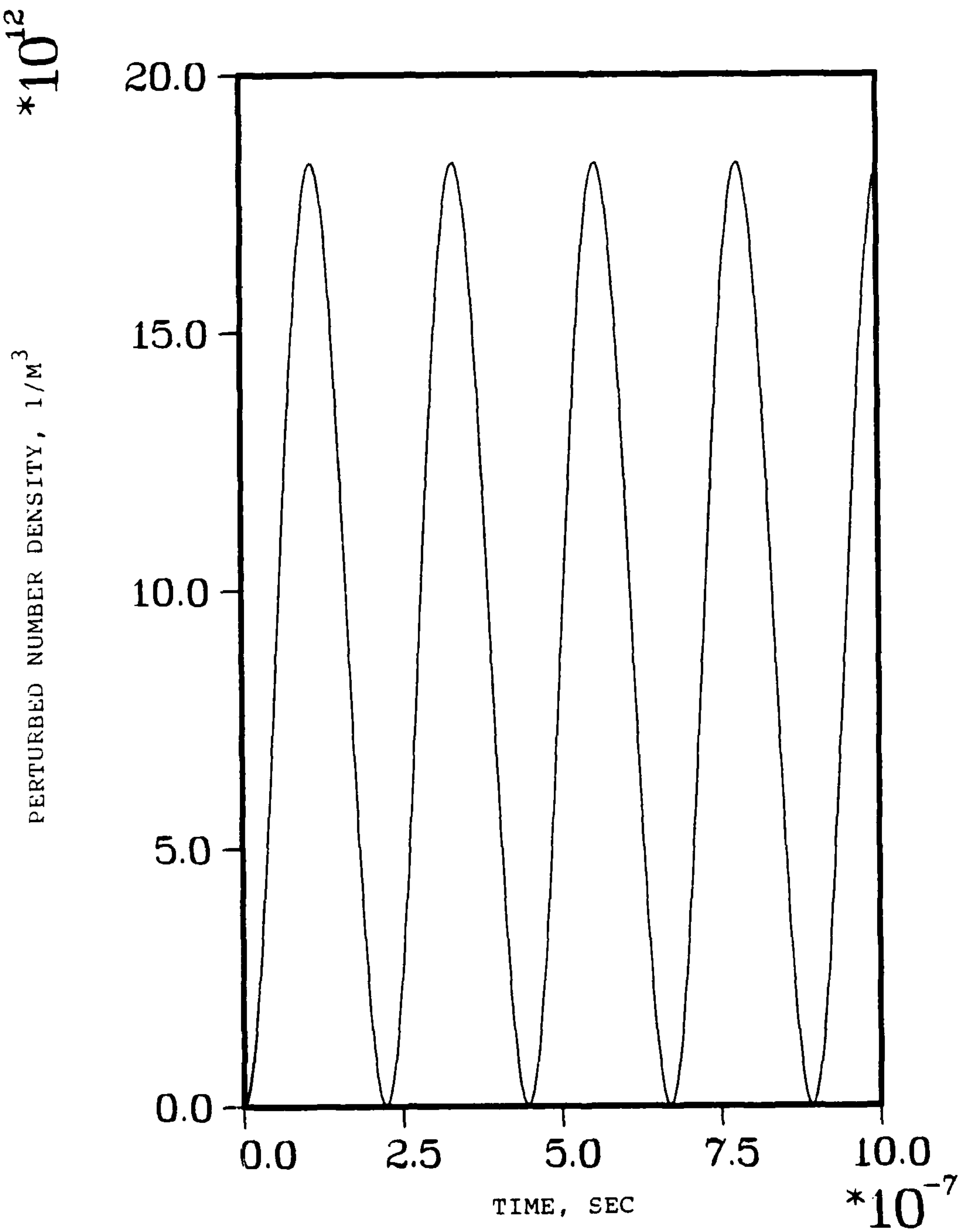


Figure 2. N(r,t)-vs-Time, Section 2-4

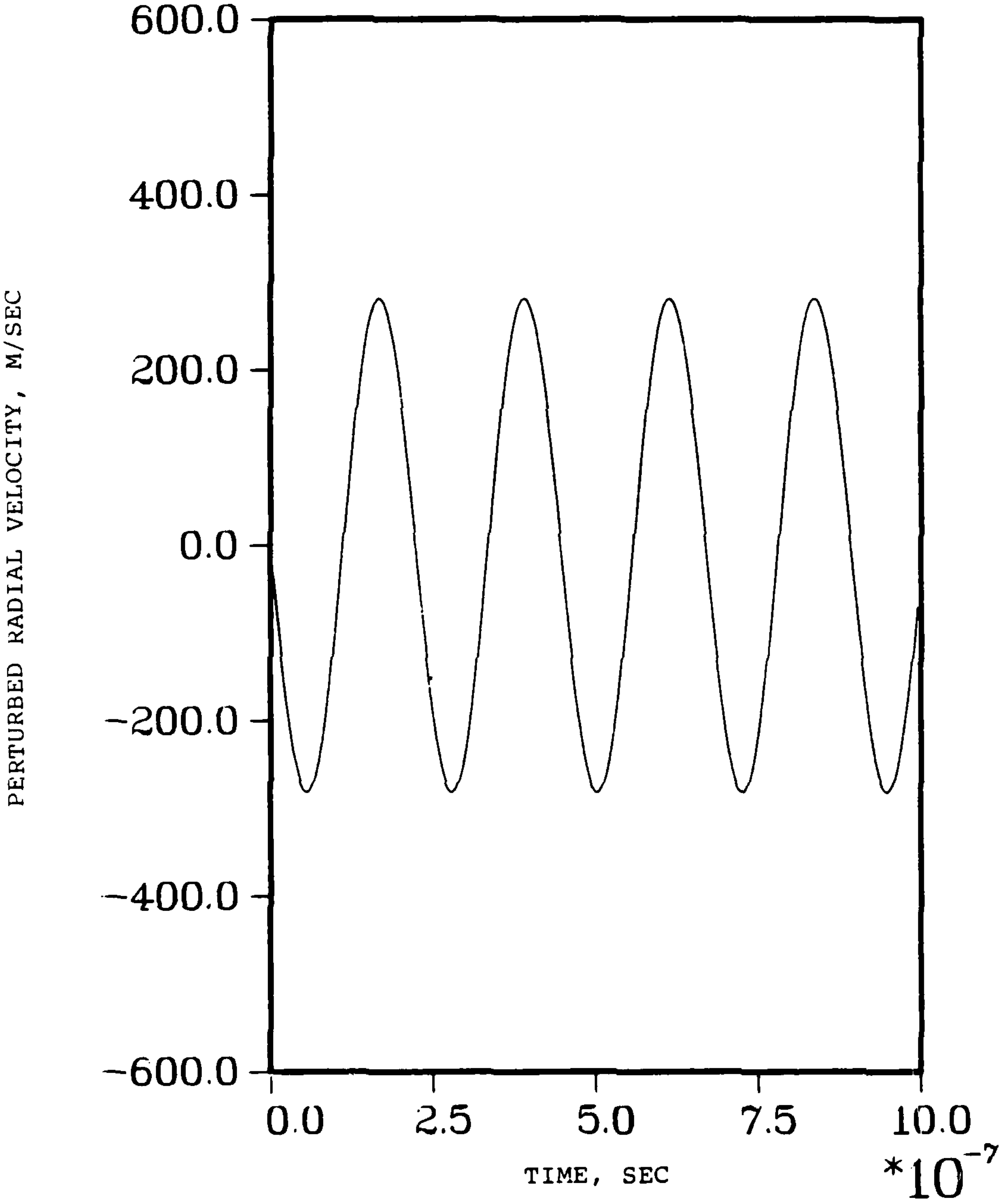


Figure 3. $V_r(r,t)$-vs-Time, Section 2-4

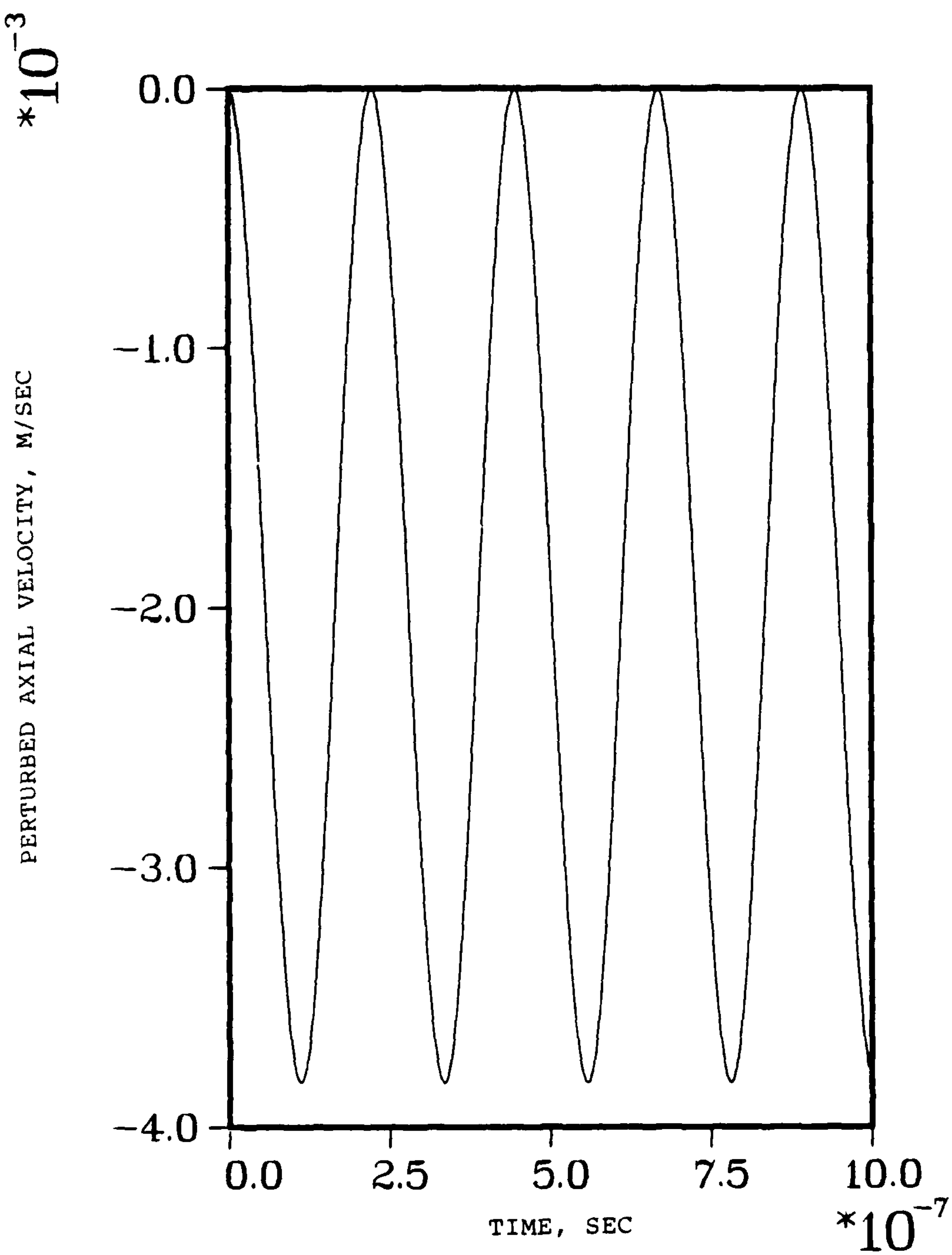


Figure 4. $V_z(r,t)$-vs-Time, section 2-4

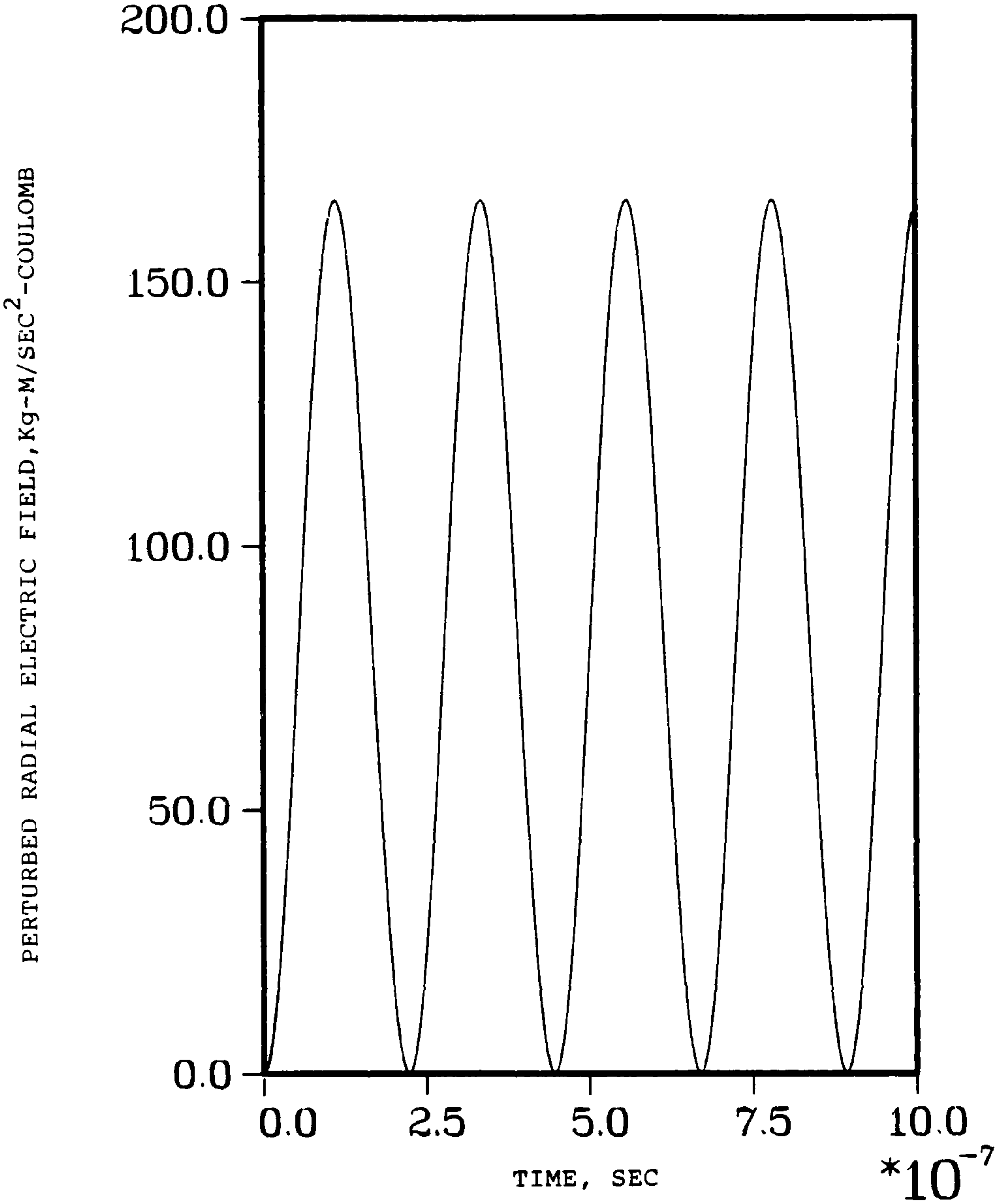


Figure 5. $E_r(r,t)$-vs-Time, Section 2-4

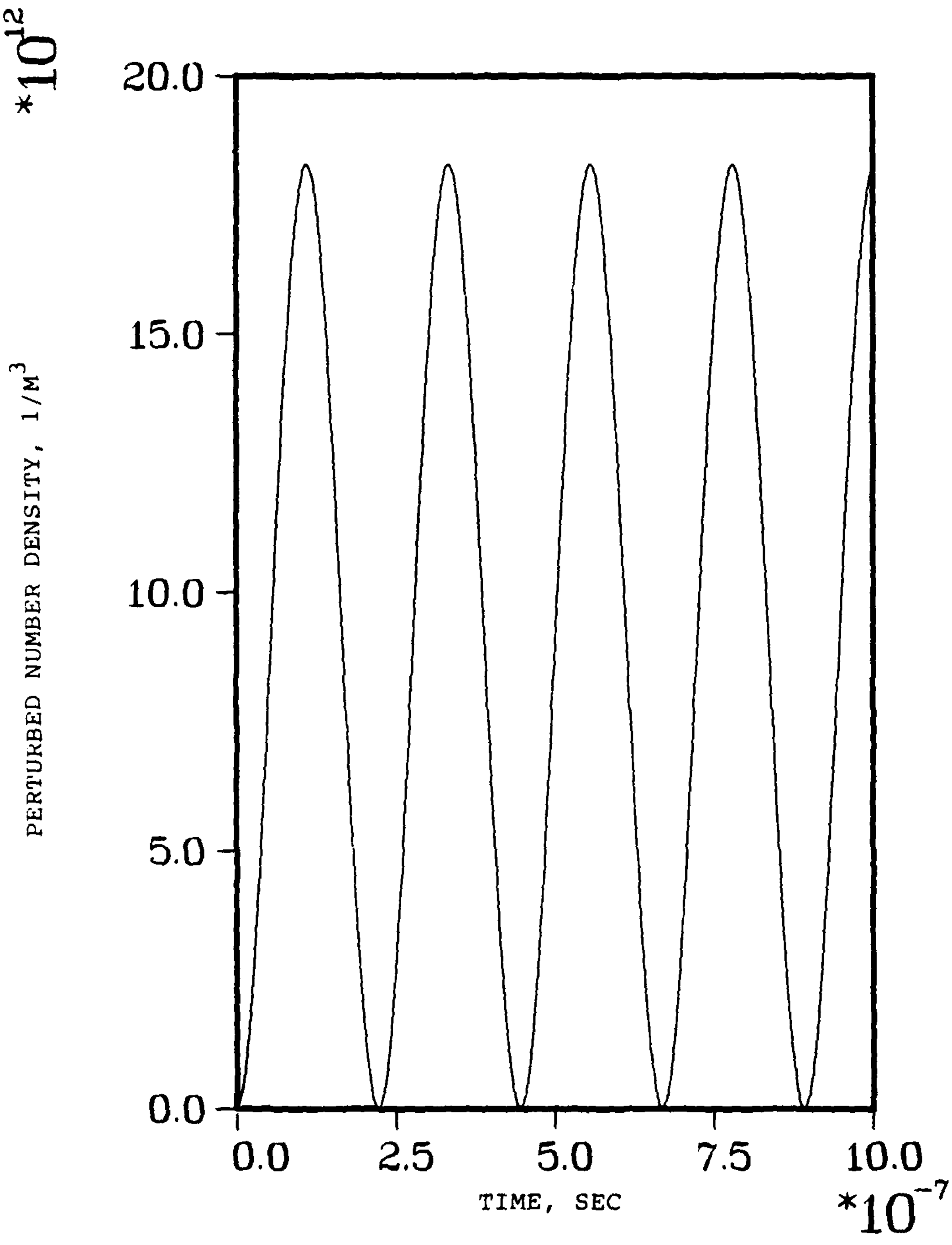


Figure 6. N(r,t)-vs-Time, Section 2-5.1

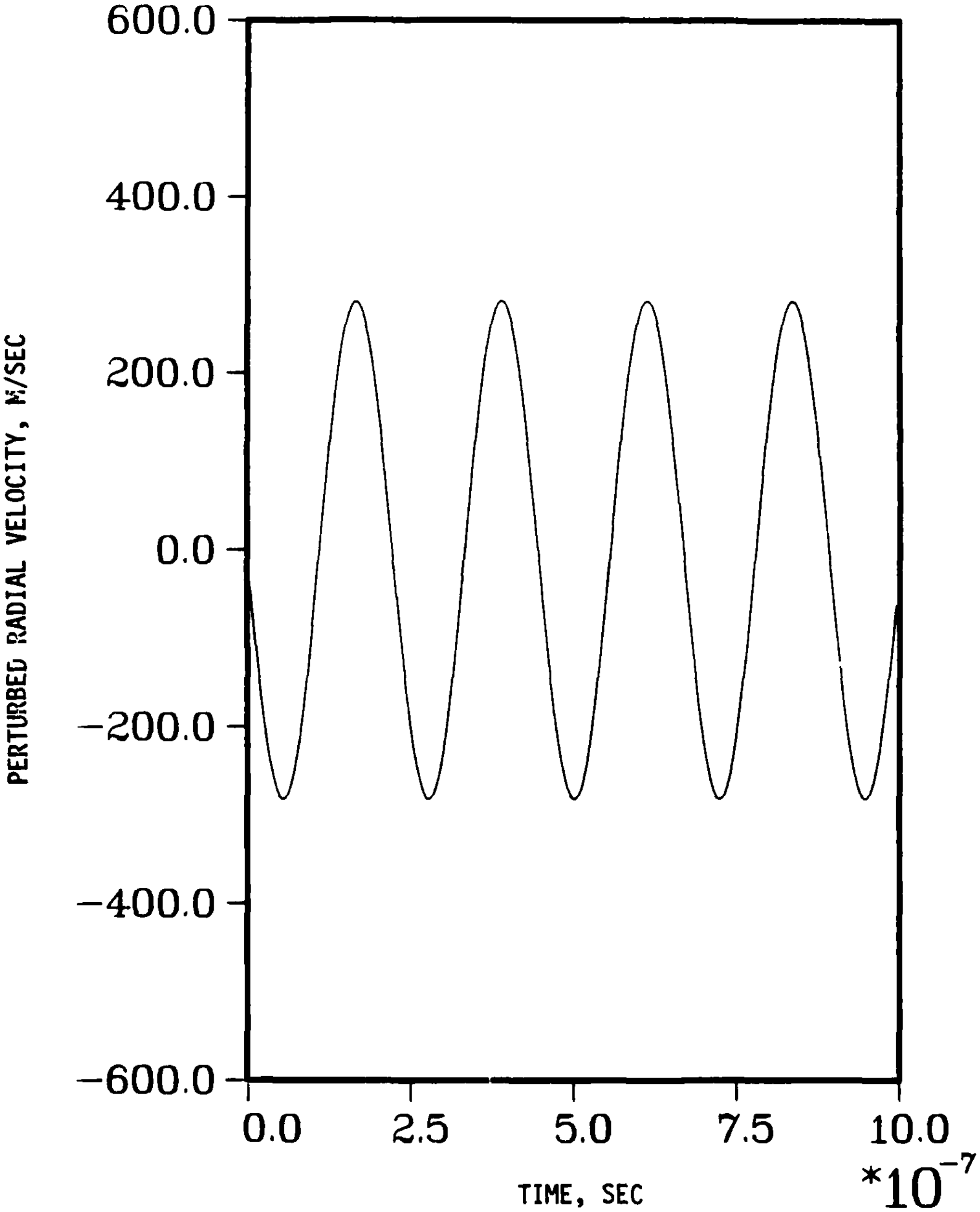


FIGURE 7. $V_r(r,t)$-vs-TIME, SECTION 2-5.1

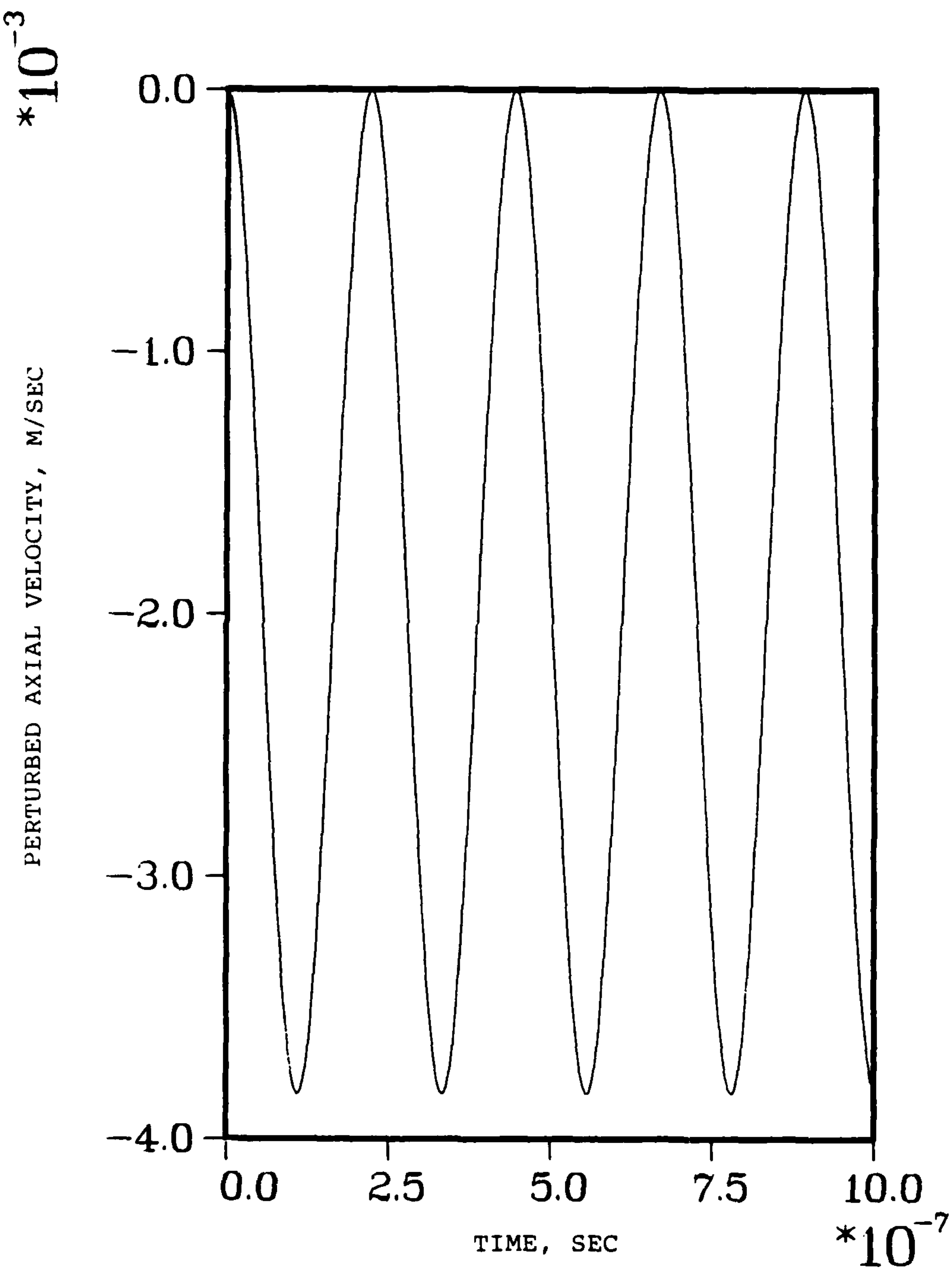


Figure 8. $V_z(r,t)$-vs-Time, Section 2-5.1

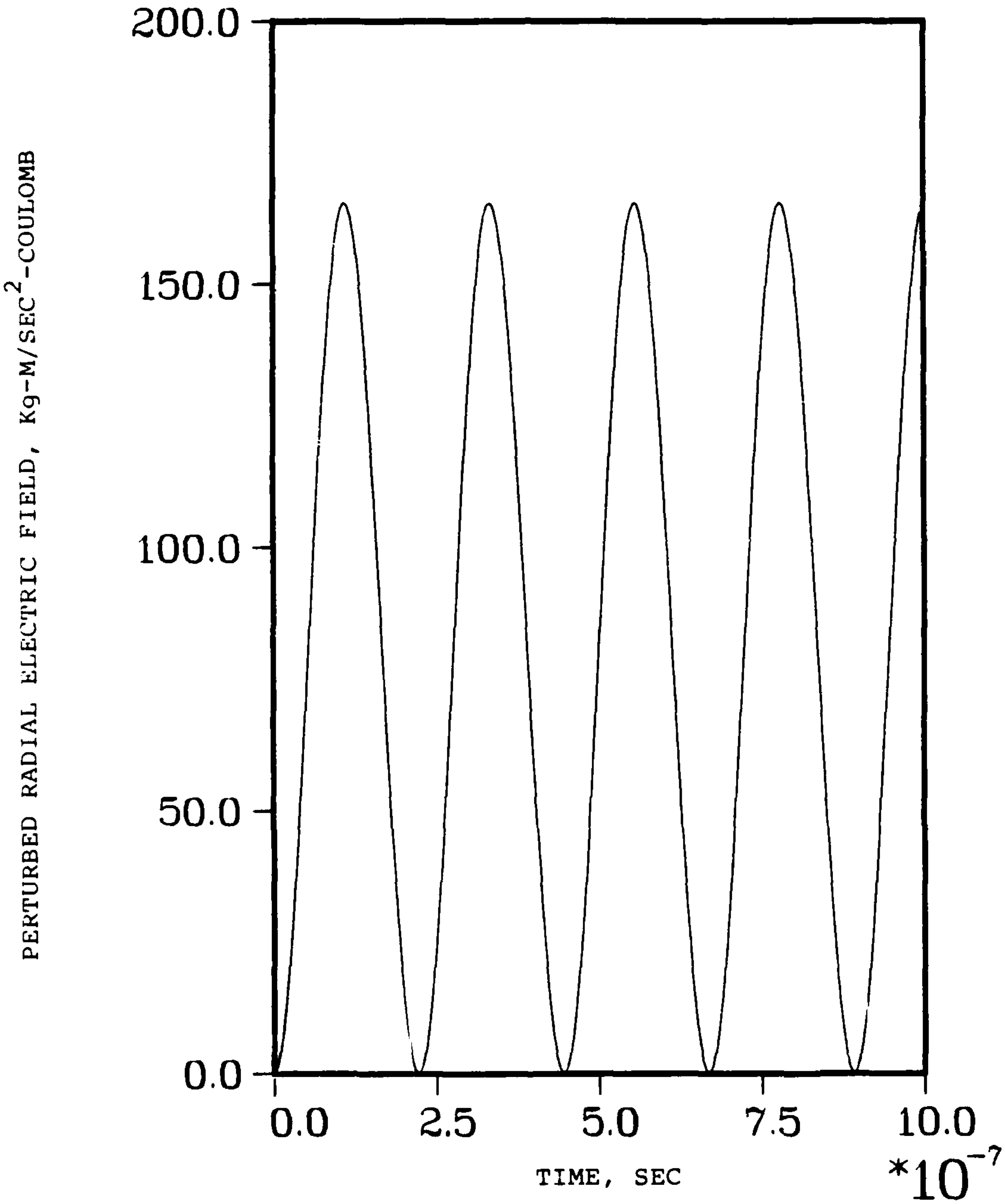


Figure 9. $E_r(r,t)$-vs-Time, Section 2-5.1

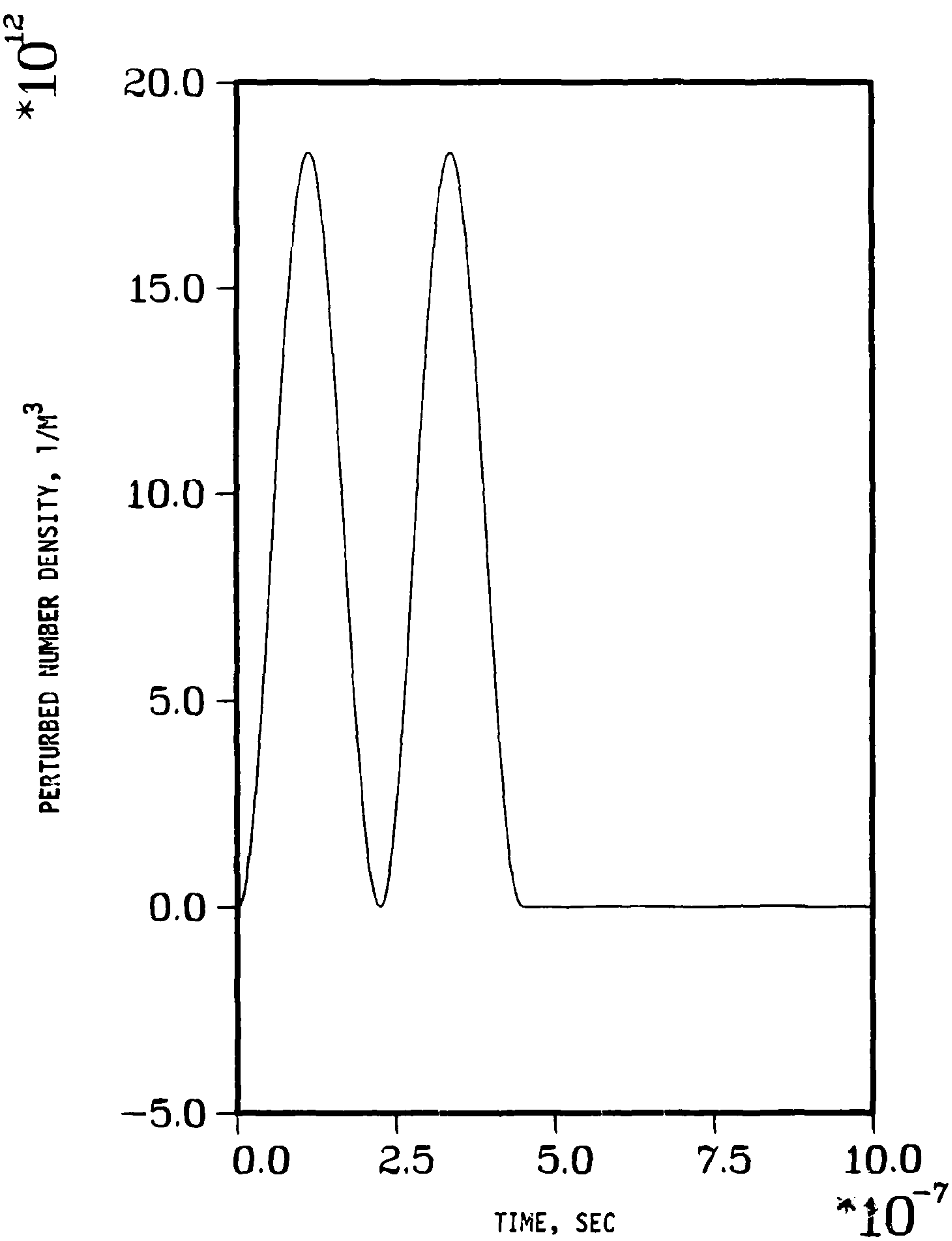


FIGURE 10. N(r,t)-vs-TIME, SECTION 2-5.2

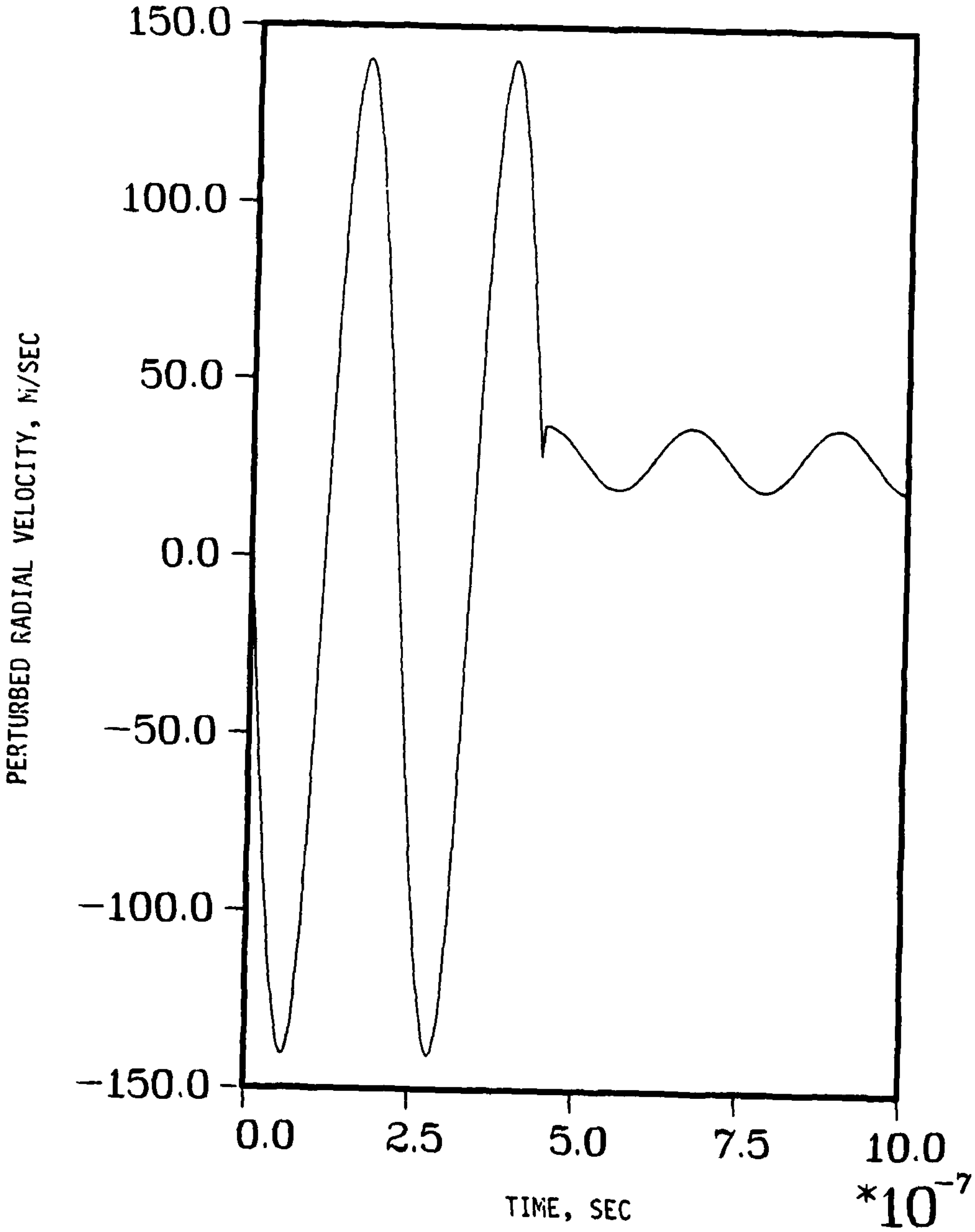


FIGURE 11. $V_r(r,t)$-vs-TIME, SECTION 2-5.2

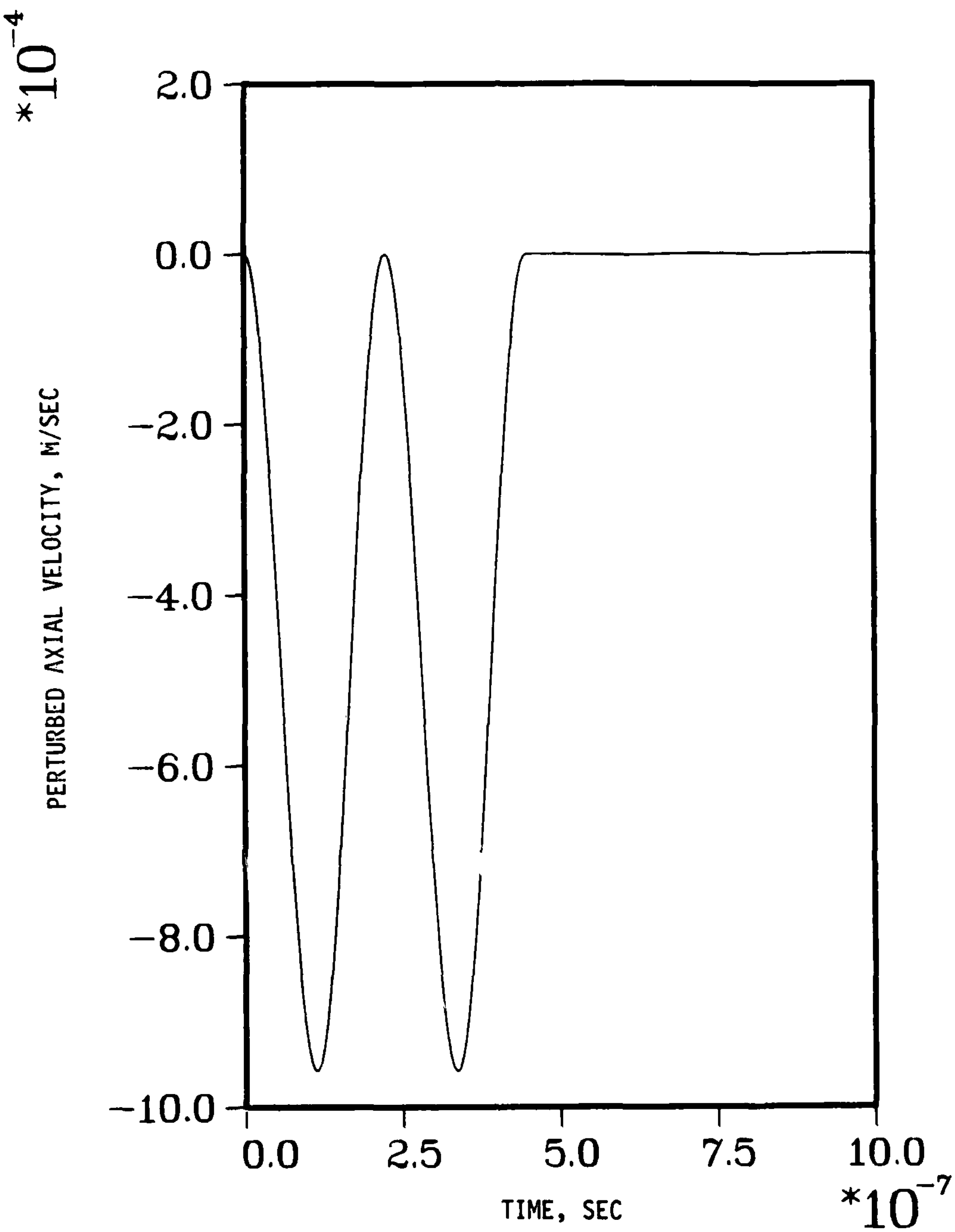


FIGURE 12. $V_z(r,t)$-vs-TIME, SECTION 2-5.2

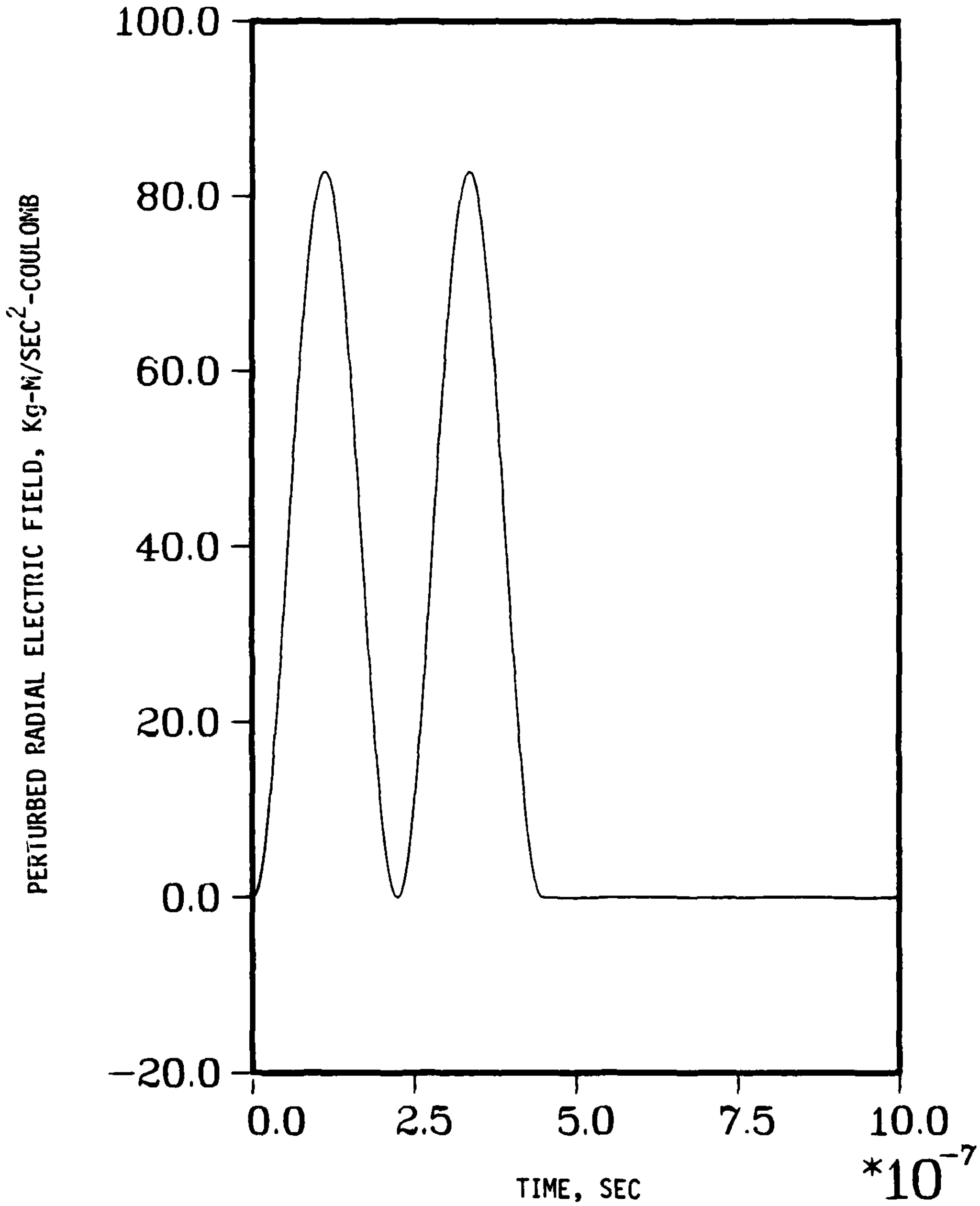


FIGURE 13. $E_r(r,t)$-vs-TIME, SECTION 2-5.2

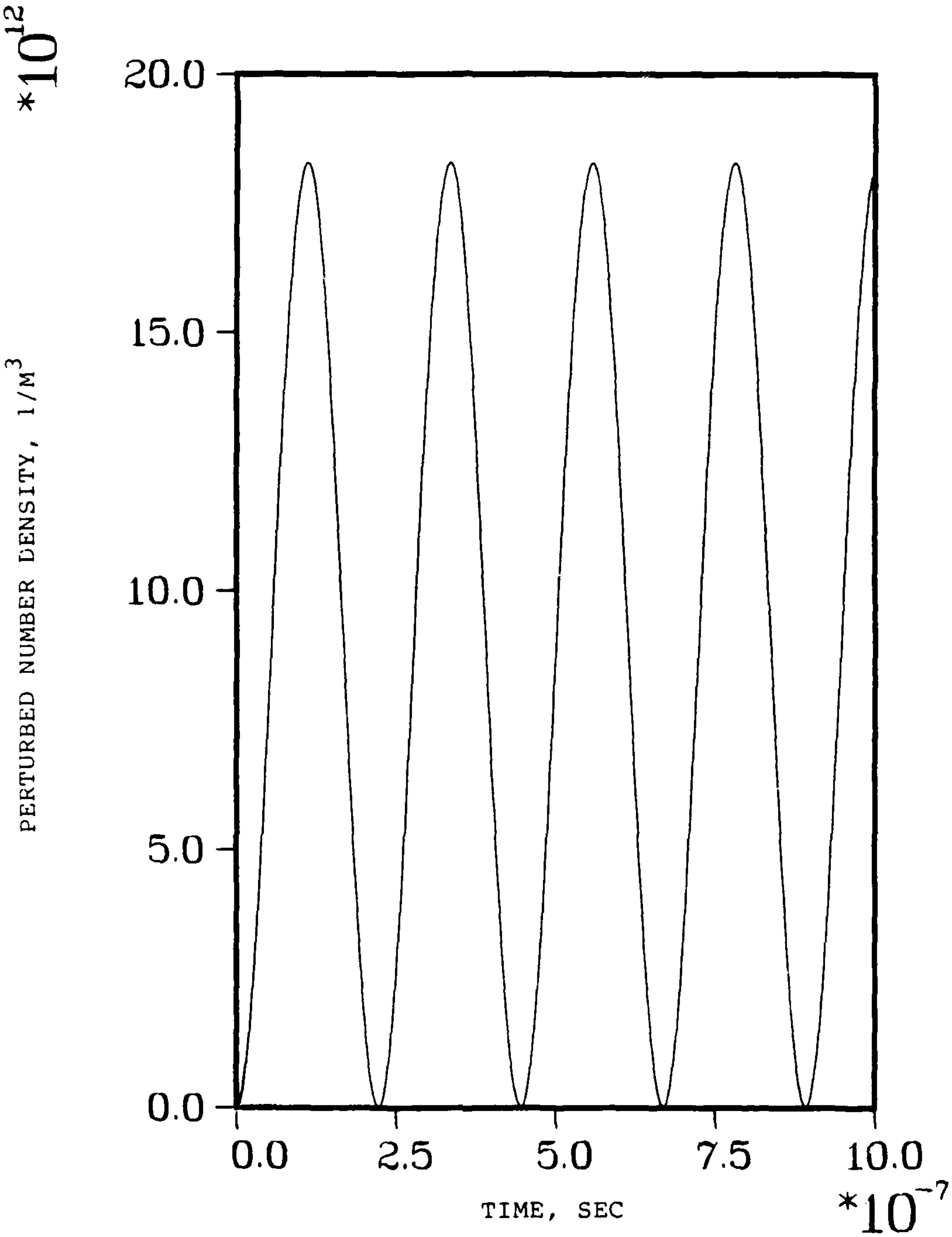
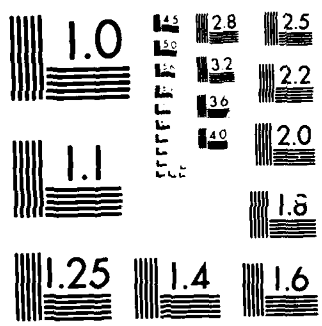


Figure 14. N(r,t)-vs-Time, Chapter 3

AD-A164 041 NUMERICAL SOLUTIONS OF SELECTED PROBLEMS WHICH ARISE IN THE THEORY OF PAR..(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI.. J A MOBLEY 2/2

UNCLASSIFIED DEC 85 AFIT/GAE/MA/85D-1 F/G 20/7 NL

END
FILMED
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

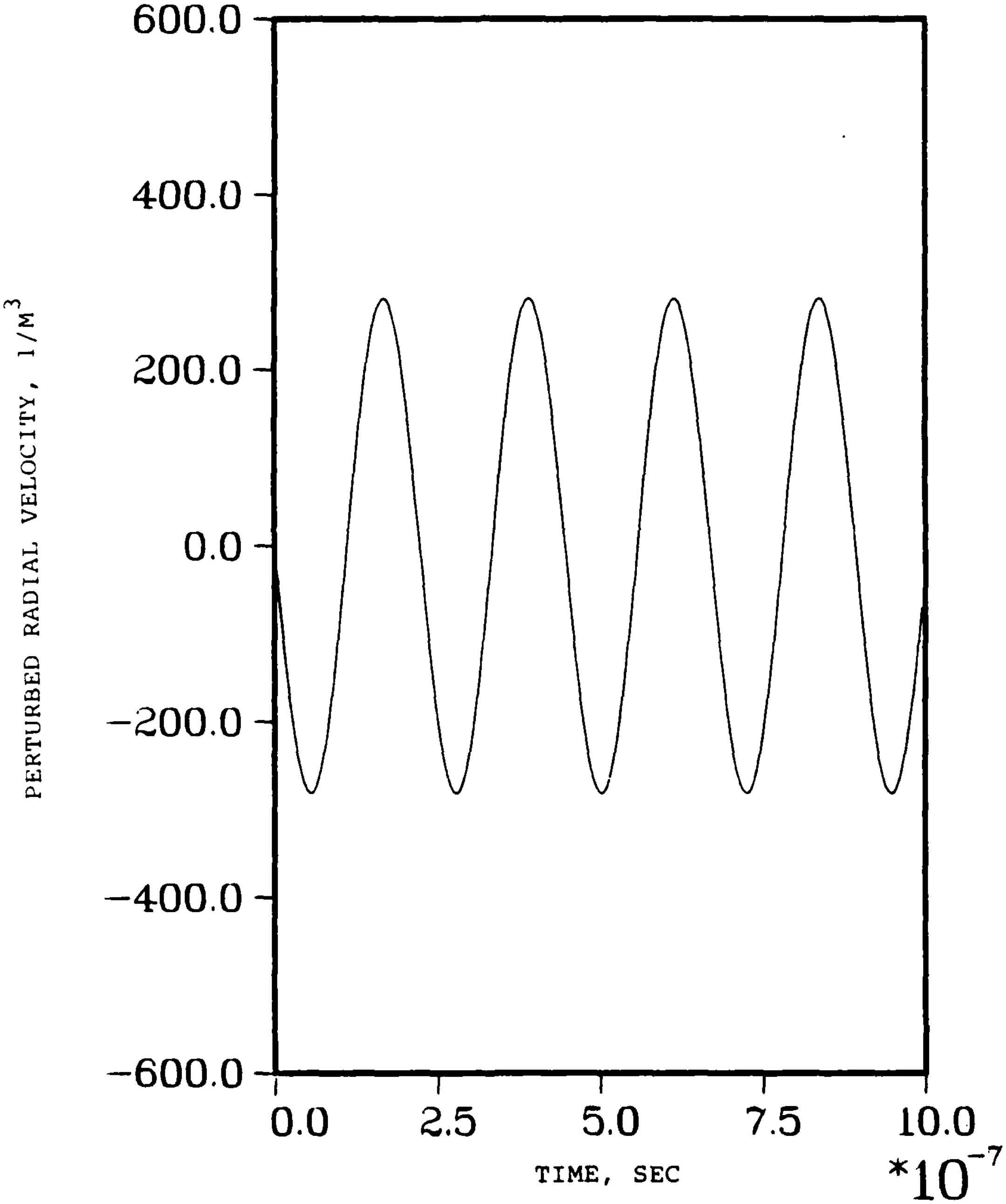


Figure 15. $V_r(r,t)$-vs-Time, Chapter 3

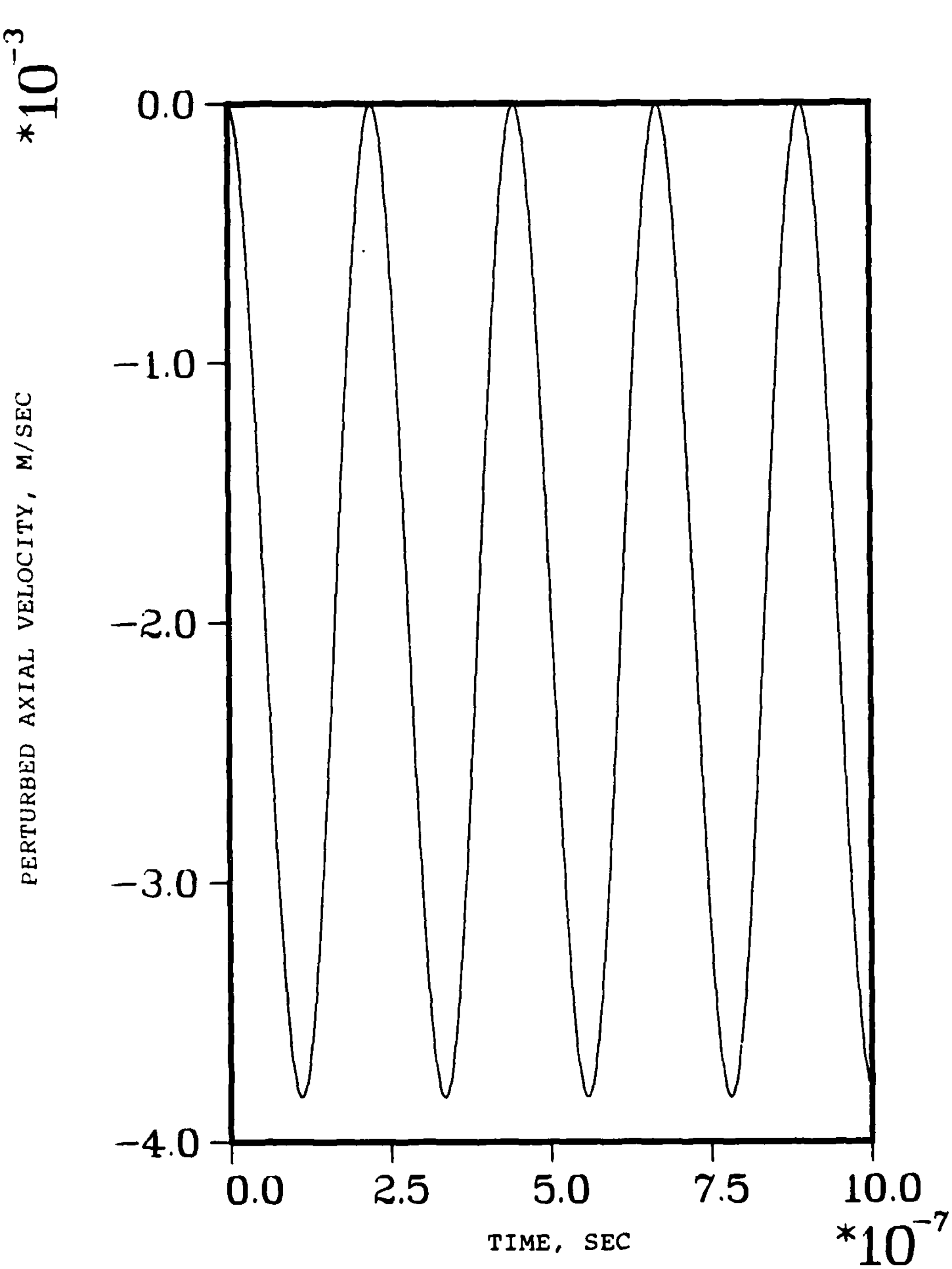


Figure 16. $V_z(r,t)$-vs-Time, Chapter 3

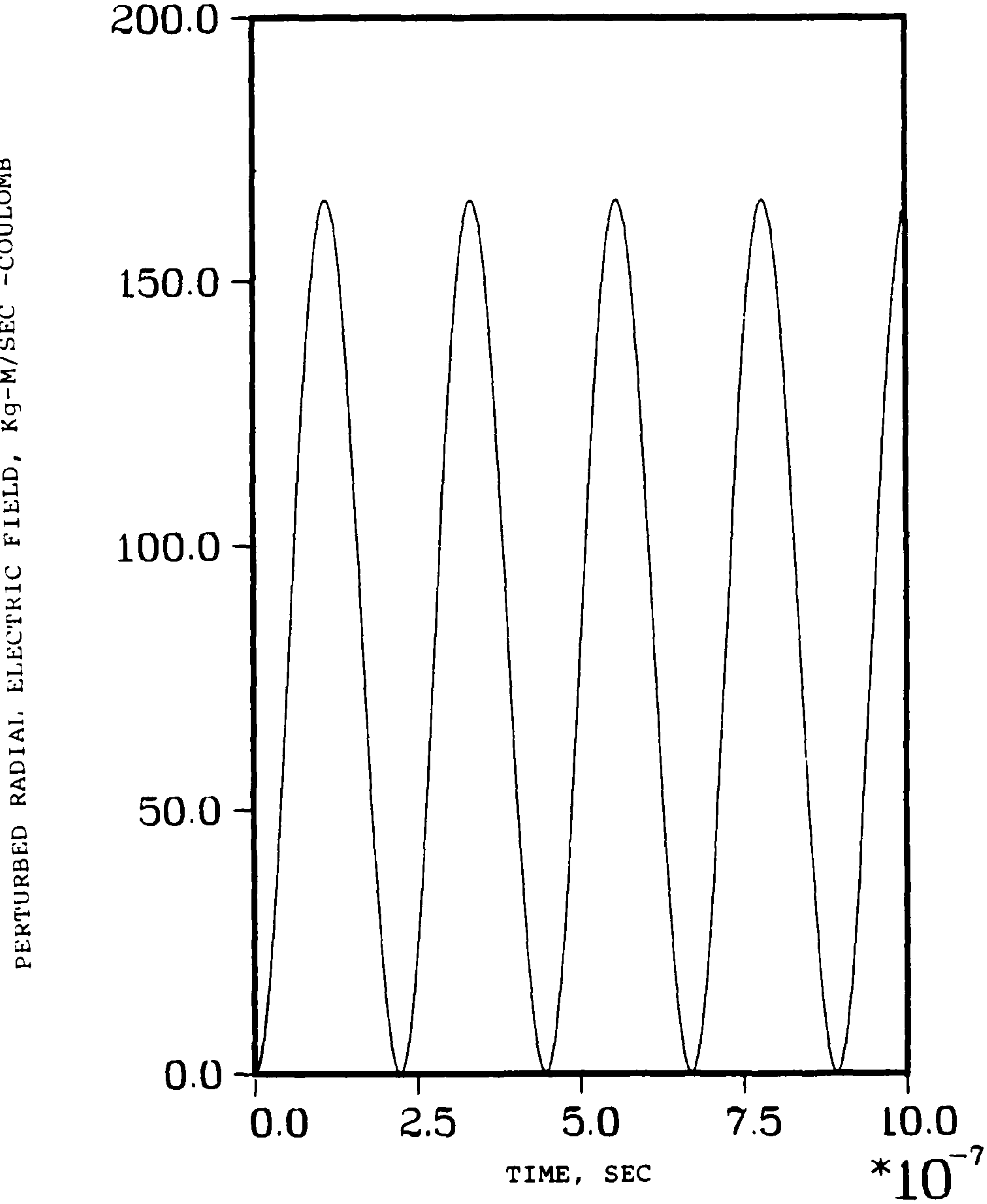


Figure 17. $E_r(r,t)$-vs-Time, Chapter 3

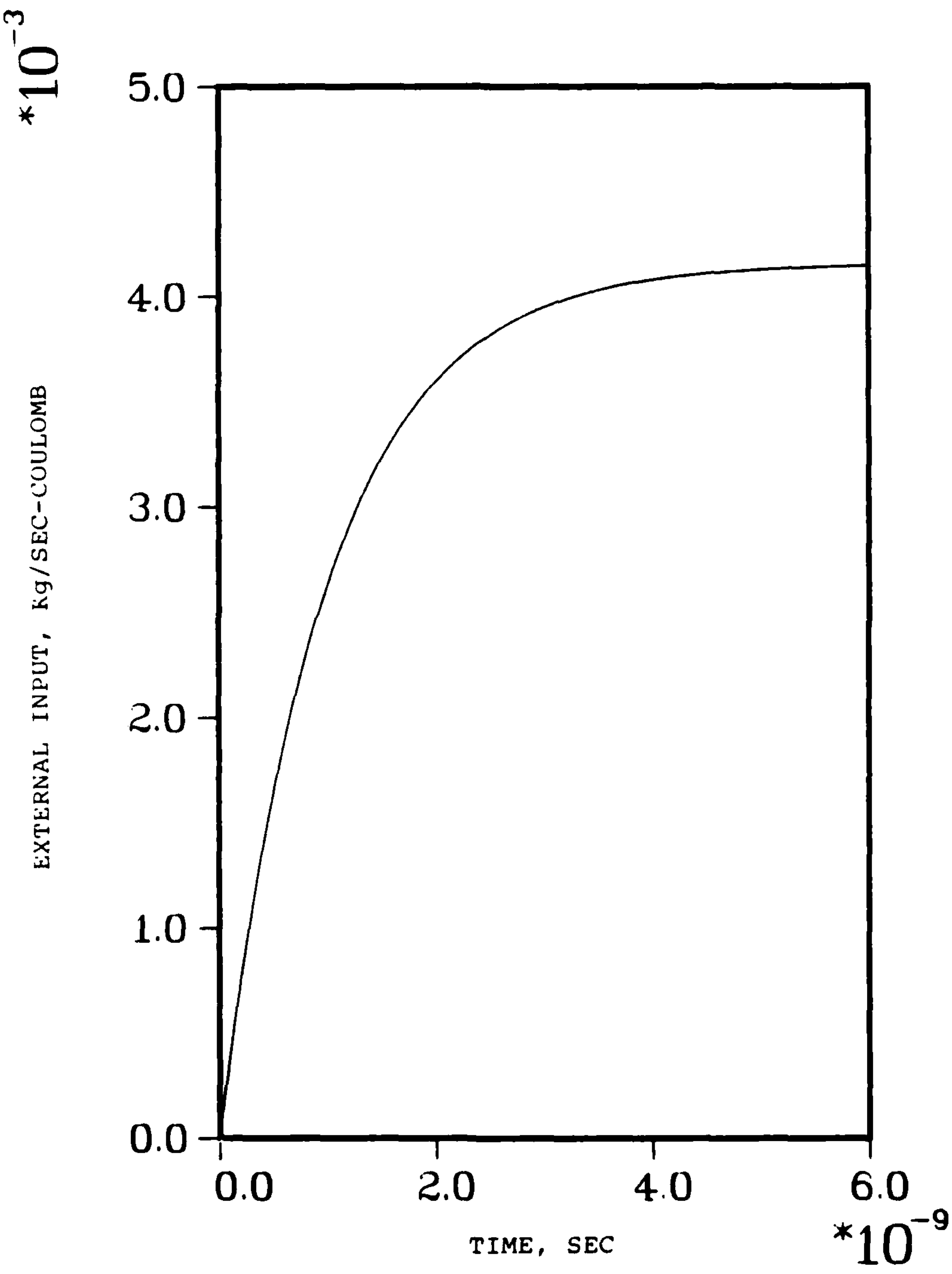


Figure 18. External Input-vs-Time

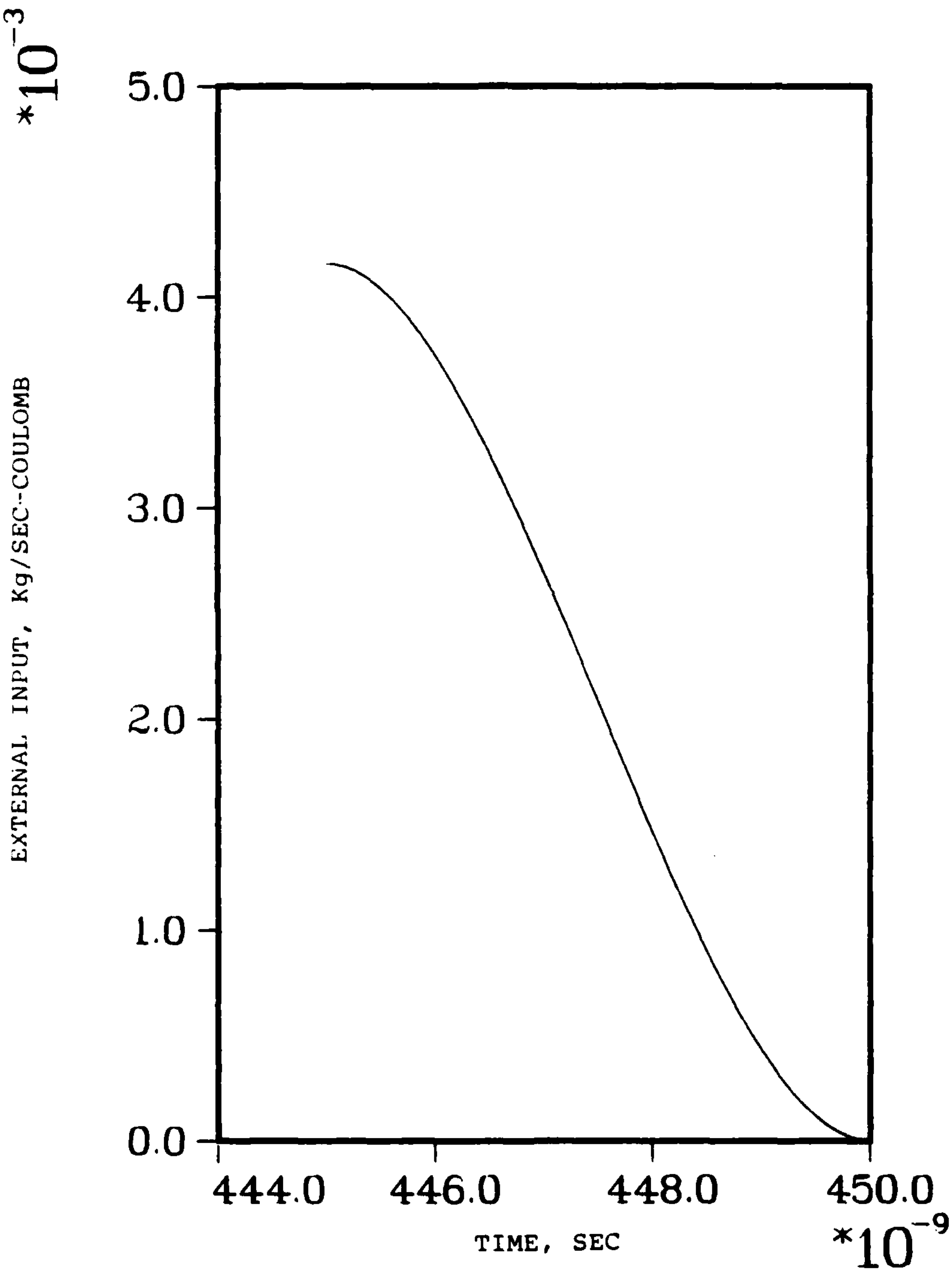


Figure 19. External Input-vs-Time

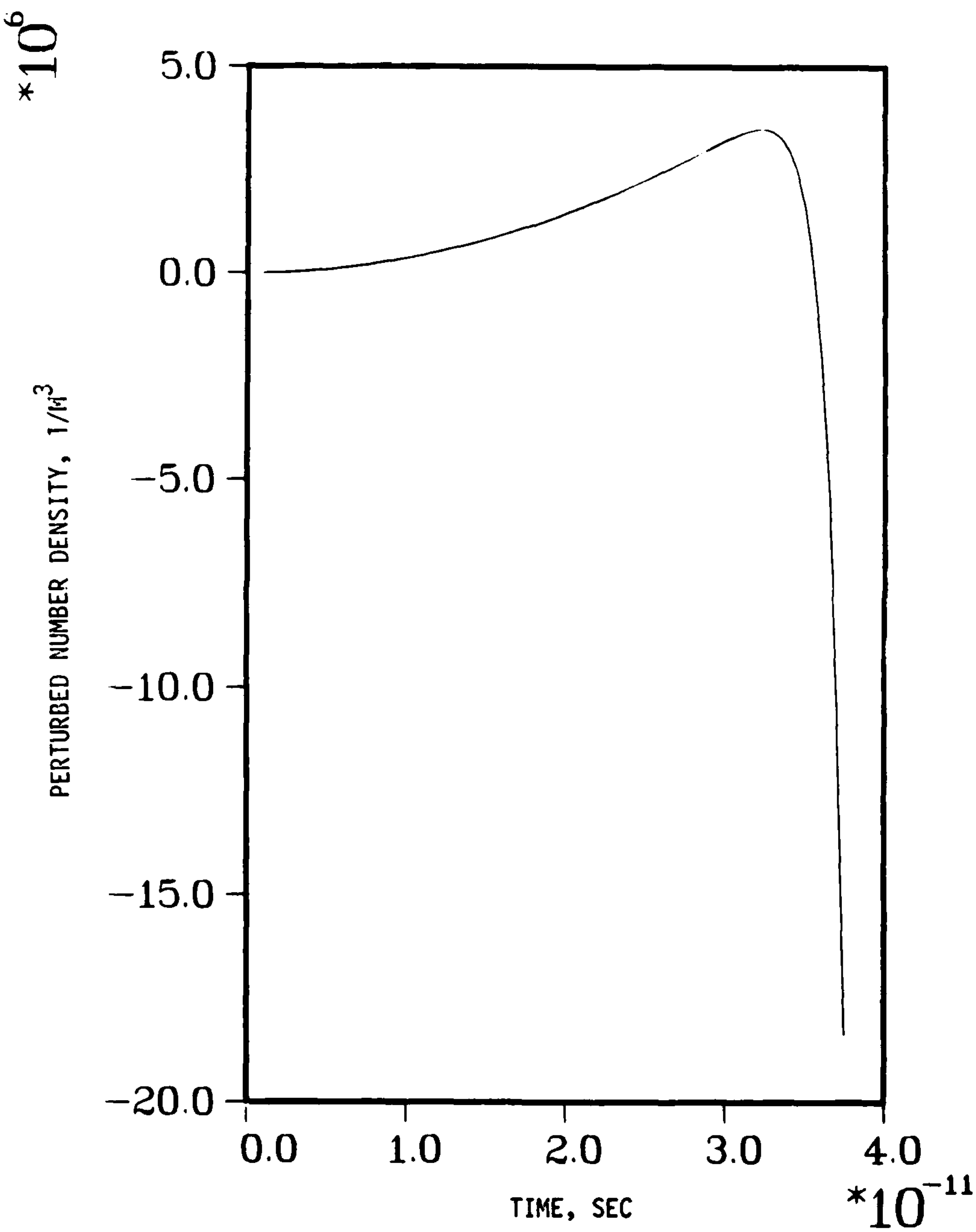


FIGURE 20. N(r,t)-vs-TIME, CHAPTER 4

Bibliography

1. Stafford, Major Max A. Analysis of the Dynamic Behavior of an Intense Charged Particle Beam Using the Semigroup Approach. PhD dissertation. School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB OH, June 1985

2. McKee, Lt Col Leslie W. Accelerator Equillibrium. Unpublished report. School of Engineering, Air Force Institute of Technology (AU), Wright-Patterson AFB OH, 1985.

3. Bueche, Frederick J. Theory and Problems of College Physics (Seventh Edition). New York:McGraw-Hill Book Company, 1979.

4. Isaacson, Eugene and Herbert B. Keller. Analysis of Numerical Methods. New York:John Wiley and Sons, 1966.

5. Lawson, J. D. The Physics of Charged-Particle Beams. Oxford:Clarendon Press, 1978.

VITA

Captain James A Mobley jr. was born on 14 April 1957 in Mount Holly New Jersey. He graduated from high school in 1975 in Titusville, Florida. In June 1980, he graduated from Auburn University with a Bachelor of Aerospace Engineering degree, and was Commissioned through the Air Force Reserve Officer Training Corps. From 1982 to 1984, he served as a systems engineer at the Air Force Operations Test and Evaluation Center, Patrick AFB Florida. Captain Mobley entered the Air Force Institute of Technology in May 1984 to pursue a Masters of Aeronautical Engineering degree.

Permanent Address:
1448 Wakefield Terr.
Titusville, FL 32796

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) AFIT/GAE/MA/85D-1	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION School of Engineering Air Force Institute of Tech 6b. OFFICE SYMBOL (If applicable) AFIT/EN	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State and ZIP Code) Wright-Patterson AFB, Ohio 45433 Air Force Institute of Technology	7b. ADDRESS (City, State and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State and ZIP Code)	10. SOURCE OF FUNDING NOS. PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT NO.
11. TITLE (Include Security Classification) See Box 19	

12. PERSONAL AUTHOR(S)
James A. Mobley, Jr. BAE, Capt., USAF

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
MS Thesis	FROM ______ TO ______	1985 December	110

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	Closed Form Solution; Discrete Solution; Varying Inputs

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

NUMERICAL SOLUTIONS OF SELECTED PROBLEMS WHICH ARISE IN THE THEORY OF PARTICLE BEAMS

Thesis Advisor: Dr. Dennis Quinn

Approved for public release: IAW AFR 190-1.
16 JAN 86
LYNN E. WOLAVER
Dean for Research and Professional Development
Air Force Institute of Technology (ATC)
Wright-Patterson AFB OH 45433

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21. ABSTRACT SECURITY CLASSIFICATION
UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐	UNCLASSIFIED

22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE NUMBER (Include Area Code)	22c. OFFICE SYMBOL
Dr. Dennis Quinn Professor of Mathematics	513-255-7210	AFIT/MA

DD FORM 1473, 83 APR EDITION OF 1 JAN 73 IS OBSOLETE.

SECURITY CLASSIFICATION OF THIS PAGE

Abstract

A previous study at the Air Force Institute of Technology has led to the development of a closed form solution to the dynamic behavior of a charged particle beam. This solution was found for a simple, but nontrivial, single degree of freedom model (the "electrostatic approximation model"). This ~~study~~ investigates the characteristics of this closed form solution using three different types of inputs. The closed form solution is then verified by using finite differences to generate compatible results. Also investigated are the characteristics of the dynamic model prior to the implementation of the electrostatic approximation.

END

FILMED

3-86

DTIC